Problem 5 (due Monday, November 3)

Two \$n\$ by \$n\$ matrices \$M,N\$ with real entries are called strongly independent if the matrix \$al+bM+cN\$ is invertible for any real numbers \$a,b,c\$ which are not all \$0\$.

- a) Show that if \$MN=NM\$ then \$M,N\$ are not strongly independent.
- b) For any \$n\$ which is a multiple of 4 construct two strongly independent matrices of size \$n\$.

(Here \$1\$ denotes the identity matrix.)

Three solutions were submitted: by Ashton Keith (Purdue University), Gerald Marchesi, and Alif Miah (a partial solution). For a detailed solution, a generalization of the problem and its connections to some deep results in topology see the following link Solution.

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