Problem 5 (due Monday, November 8)

We call a positive integer N **prosperous** if $\ln(N)+\sin(N)=2(N+1) \$ \\ \phi(N)\sigma(N)= (N-5)(N+3).\] Knowing that both N and N-504\$ are prosperous, find N.

Remark. Here \$\phi\$ is the Euler function and \$\sigma\$ is the sum of divisors:

 $^{\cdot}_{1}$ \$\phi(N) =\$ the number of positive integers which are relatively prime to \$N\$ and do not exceed \$N\$,

| \$\sigma(N) = \$ the sum of all positive divisors of \$N\$.

I These functions are studied in elementary number theory (a topic of Math 407). They both are so called **multiplicative functions**: for any two **relatively prime** integers \$M,N\$ we have f(M)=f(M)f(N), where \$f\$ is either \$\phi\$ or \$\sigma\$.

We received solutions from Ashton Keith, Maxwell T Meyers, and Pluto Wang. Ashton provides a short solution which requires some direct computations at the end. Maxwell shows that the first condition in the definition of a prosperous number holds for \$M\$ if and only if \$M=pq\$ is a product of two distinct prime numbers. From this he concludes that \$M\$ is prosperous if and only if \$M=p(p+4)\$ where both \$p\$ and \$p+4\$ are prime numbers. From this he shows that \$N=2021\$ is the only solution to the problem. Pluto has a partial solution, which proves what Maxwell showed but only when \$M\$ is a product of distinct prime numbers (i.e. \$M\$ is square-free). Maxwell's solution is the same as my original solution. However, I later realized that the second condition in the definition of the prosperous number implies the first one. For more details see the following link Solution.

From:

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Permanent link:

https://www2.math.binghamton.edu/p/pow/problem5f21

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