Problem 4 (due Monday, March 25)

A function $f:\mathbb R^2\longrightarrow \mathbb R$ has the following properties:

a) the partial derivatives $\displaystyle \frac{\partial f}{\partial x}$ and $\displaystyle \frac{\partial f}{\partial y}$ are continuous on $\mathbb R^2$;

b) $\displaystyle \left (\frac{\partial f}{\partial x}(x,y)\right)^2+\left (\displaystyle \frac{\partial f}{\partial y}(x,y)\right)^2\leq \frac{\partial f}{\partial x}(x,y)$ for every $(x,y)\in \mathbb R^2$;

c) $f(x,0)=0$ for all $x\in \mathbb R$.

Prove that $f(x,y)=0$ for all $(x,y)\in \mathbb R^2$.

We received only one (partial) solution, from Beatrice Antoinette. For a complete solution see the following link Solution.