Problem 4 (due Monday, April 12)
a) Let $\$$ f:\mathbb R \longrightarrow \mathbb $\mathrm{R} \$$ be a differentiable function such that $\$ f(\backslash \sin x)=\backslash \sin f(x) \$$ for every $\$ x$ \in $\backslash m a t h b b$ R\$. Prove that if $\$ f \$$ is not identically zero then $\$ \backslash$ displaystyle $\backslash$ lim_\{x|to 0$\} \backslash$ frac $\{f(x)\}\{x\} \$$ exists and is equal to $\$ 1 \$$ or $\$-1 \$$.
b) Prove that there is a continuous function \$f:\mathbb $R$ \longrightarrow $\backslash m a t h b b ~ R \$$ such that $\$ f(\backslash \sin x)=\backslash \sin f(x) \$$ and $\$ \backslash$ displaystyle $\backslash$ lim_ $\left\{x \mid\right.$ to $\left.0^{\wedge}+\right\} \backslash f r a c\{f(x)\}\{x\} \$$ does not exist.
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Two solutions were submitted: by Paul Barber and Ashton Keith. Neither one is complete. Ashton attempts to solve part a) under additional assumption that $\$ \mathrm{f}^{\prime} \$$ is continuous at 0 . While his solution has some gaps, the ideas are very nice indeed and they can be improved to a complete solution (under the additional assumption). For more details and to see complete solutions see the following link Solution.

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