Problem 4 (due on Monday, October 24)

A sequence (a_n) of positive integers has been created using the following process: \[a_{n+1}=a_n+3\frac{a_n}{p_n}\] where p_n is a prime divisor of a_n. Prove that there is a positive integer k such that the equality $a_{n+k}=2ka_n$ holds for infinitely many values of n.

We received a solution from Garrett Gramzow, Prof. Vladislav Kargin, and Ashton Keith. All three solutions as well as our original solution are essentially following the same line of argument, though Garrett's solution needs some improvements in exposition. It turns out that either $k=1$ or $k=2$ works. For a detailed solution see the following link Solution.