Problem 3 (due Monday, March 17)

Let $f:\mathbb R\longrightarrow \mathbb R$ be an even continuous function such that $f(x+2)=f(x)$ for all x and f is increasing on $[0,1]$. Define a new function $g:\mathbb R\longrightarrow \mathbb R$ by \[g(x)=\int_{0}^{2}f(t)f(t+x)\text{d}t.\] Prove that $g(1)$ is the smallest value of g.

The problem was solved by Ashton Keith (Purdue University), Josiah Moltz, and Dr Mathew Wolak. For details see the following link Solution.