Problem 2 (due on Monday, September 26)

Find all positive integers n such that $n!$ divides $(2n+1)^{2n}-1$.
(Here $n!=1\cdot 2\cdot \ldots \cdot n$ is the factorial of n).

The positive integers in question are $1,2,3,5,6$. Our solution and the two submitted solutions all follow the same strategy: show that with a finite and small list of exceptions, the highest power of 2 which divides $n!$ is larger than the highest power of 2 which divides $(2n+1)^{2n}-1$, hence $n!$ can not be a divisor of $(2n+1)^{2n}-1$. The small number of exceptions is then handled by hand. For a complete solution and some additional problems and material see the following link Solution.