Cost, Revenue, Profit

Cost/revenue/profit 1, Cost/revenue/profit 2, Cost/revenue/profit 3

Demand function and cost function

For a little later in the unit: Marginal Revenue, Average Cost, Profit, Price & Demand Function

Compound interest, present and future value

Doubling something (after 1 year, say) is what we call 100% growth. Interest is generally paid at a much smaller amount. This video begins first with 100% growth, so the growth of the ball is easy to draw. This way the lecturer can get you to “e”. He then goes on to show a more reasonable growth rate r. First, an easy one:

Where does e come from?

Understanding the number e (exponential growth).

This last one is a good presentation of the several types of problems seen in finding present and future value when interest of an investment or loan is compounded. The lecturer uses the variable *A* for *F* for future value. For *F* I used *P*(*t*) to show the it is a function of time, and *Po* for present value.

Compound interest, present and future value problems

Limits

Finding limits from a graph (This is one of his rougher videos, but well explained.)

Evaluate limits using properties, Ex 1

Evaluate limits using properties, Ex 2

More techniques for evaluating limits, Ex 3 (gives a little jump on continuity)

Ex 4 involving radicals

Ex 5 also with radicals

IMPORTANT Infinite limits in which a function goes to positive infinity or negative infinity as x approaches a:

Ex 6 involving rational expressions

Optional, good insight, might actually help you better understand the actual limits we have done:

Precise definition of limit

Continuity

Main idea: A function *f*(*x*) is continuous at *x* = *a* if *f*(*a*) exists and if limit of *f*(*x*) as *x* approaches *a* is equal to *f*(*a*).

First, check these helpful videos on graphing piecewise functions by Patrick, if you need them:

Graph a piecewise fcn 1 and Graph a piecewise fcn 2

Patrick discusses limits and the relationship to continuity. Watch for the important ideas, as we will discuss at length.

Continuity and limits made easy

Discontinuities in a function (piecewise)

More inspecting for discontinuities of a piecewise function

Derivatives

Instantaneous rate of change of a function at point via the difference quotient (DQ)

The numerical slope of a tangent line at some point of a function is *derived* from the function itself by means of the *difference quotient*. The function that describes the behavior of the slope of the tangent line at *any* point along the graph of a function is called the “derivative function” (or simply, the “derivative”).

This function we will soon see is the *marginal cost function*.

We evaluate a derivative function at a given x (say, $x = a$), we find the instantaneous rate of change of the function at that point.

This is an essential video: Difference quotient (DQ) and the definition of derivative

Here is the process: Finding derivative with DQ, Ex 1

and Finding derivative with DQ, Ex 2

Here are examples for finding equation l of line tangent to f(x) at a given point:

Ex 1, Ex 2, Ex 3 and Ex 4

Longer example of finding equation of tangent line

A practical application from the laws of physics (motion): Relationship between displacement, velocity and acceleration

The Derivative Rules and examples

Recall Shortcuts to the derivative

The following are short, understandable proofs of the derivative rules:

Proof of product rule

Proof of quotient rule

Derivative of an exponential fcn with base a

Derivative of a log fcn with base a

Proof of chain rule The lecturer uses Leibniz notation dy/dx. There's a little fudging where on u(x) and g(x) being the same, but he says so.

Product rule examples

Quotient rule examples

Chain rule explained by Patrickmjt

Ex of chain rule for radical function

Ex of chain rule for natural log function

Many great examples of chain rule involving ln[u(x)]

A couple more

While you are not responsible for the proofs, only the rules, you ought to see where the formulas come from. They rely on d/dx notation, implicit methods, and log properties, as well as limit process. None of these are difficult. They are short and very clear.

Proof of derivative of exponential function (base e)

This proof requires a result from a proof that uses a trig graph (i.e., not in the 'scope' of this course, but not hard either.

Proof of derivative of natural log function

Proof of derivative of exponential function (base a)

Implicit differentiation

Implicit differentiation method often lets us find rates of change of one variable with respect to another even if there is no explicit function present. For example, the circle isn't a function, but its tangents are of interest to us, especially where they fail to exist. Differentiating the equation without solving for either of its branches (top or bottom semicircle) is handy using ID.

Implicit differentiation (ID)

More ID examples

(Go to 3:30 for example that involves non-trig equation)

Related rates

Real-world related rate problems are best introduced with problems of basic geometry. Clearly, physical parameters of plane figures (rectangles, triangles, circles) and solid figures (boxes, cylinders, spheres) are related by formulas. As such, as one parameter changes, then so do the other parameters. More to the point, the *rate* of change of one parameter affects the rate of change of another.

Implicit differentiation comes into play when we encounter a situation in which dimension 'a' changes with respect to another dimension 'b', and both 'a' and 'b' change with respect to time 't.' 'a' changes with respect to 't' through 'b'.

For example, the area of a circle is a function of its radius; if radius changes with time (i.e., radius is a function of time), then the rate at which the circle's area changes is also a function of time. We end up differentiating area (an *explicit* function of radius) *implicitly* with respect to time. The videos demonstrate this.

Here's a nice video by Krista, who reviews implicit differentiation before doing a related rate problem. You can't go wrong looking her up when you seek video explanation. Like Patrick, she is clear, no frills, and her conveyance of mathematics methods is great!

Implicit differentiation and related rates

Related rates 1: Area of circle and changing radius rate

Related rates 2: Area of triangle and changing side length rate

Related rates 3: Ladder sliding down the wall problem

And finally, a video on related rates that applies to a business application:

Cost and profit with respect to time

Critical numbers

Finding critical numbers is the preliminary step to using first and second derivative tests, used to examine where functions are attaining their local extremes, and the intervals on which they are increasing, decreasing. We use these skills to sketch the curves of functions and so examine what kind of behavior a function models.

A critical number f(x) is that value of x (call it 'c') in the domain of f where EITHER f'(x) = 0 OR where f'(x) does not exist.

Finding critical numbers of a fcn

There are many more videos on the HW page for this topic.

First and second derivative tests

First derivative test

Second derivative test and concavity

Curve sketching with calculus

Examples first and second derivative test to graph functions

Graphing a polynomial

Graphing a rational function

Graphing a simple rational function

Graphing another rational function

Optimization

Optimization problems dealing with geometry:

Fence problem 1

Fence problem 2

Box problem

More optimization: Wherein n = number of price reductions or increases. Notice I prefer n to the book's x for this variable, as we are used to x being quantity of sales.

Computer software sales

There are two examples done on this video. The first should be enough, and I like that it gives a graph as I did for the lamp sales problem today. The whole idea of graphing the upside down parabola and asking what its max and its y-intercept indicate is very important. (By the way, on the video's first problem, the y-intercept shown is incorrect, as it is definitely not the case that n = 0 price reductions would mean $0 revenue; but more about that in class).

Here's a similar one.

The hot dog problem

Finally, one where we have to come up with the price function (called the “demand function” here), to create the revenue function:

Optimizing revenue given two points of data

Multivariate functions

Multivariable functions up to 9:45

Partial differentiation

Partial derivatives

Local max and min of a function of two variables f(x,y)

In three-space, we find critical points essentially same as we do in two-space, but to determine if they are max, min, or saddle points, *we have no first derivative test*. Rather, we go straight to a second derivative test after finding critical values.

As you watch this be aware of the following:

1. Calculation of partial derivatives

2. Solving for critical numbers, which generally entails solving a simply linear system, but sometimes a non-linear one, and even discarding some values (Theorem 28.1).

3. Employing the SDT for f(x,y) (Theorem 28.2).

Critical points and second derivative test for local max and min of multi-variable function f(x,y)

Lagrange multipliers for solving problems in optimization with a constraint:

Prof. Kumar shows Lagrange multiplier method of solving optimization w/constraint

Lagrange Method with Krista Ex 1 and Lagrange Method with Krista Ex 2

Elasticity

Elasticity of Demand

This is one of the best mini-lectures I have seen.

The lecturer mentions the curves that have to be drawn 'many times in the course.' We won't have to draw many curves, except as we discuss the meaning.

Antiderivatives (indefinite integrals)

Antiderivatives and indefinite integration

Examples of basic indefinite integration

Antiderivative with initial conditions (finding a particular F(x))

Displacement, velocity, acceleration example

u-substitution

Examples of simple substitution

Integration by parts

Fundamental Theorem of Calculus (FTC) and Definite Integrals

The area under the curve f(x) represents the *accumulation* of that function on a stated interval.

Why this is the case is supported by the development of the FTC, seen here:

Fundamental Theorem of Calculus, part I

Fundamental Theorem of Calculus, part II (the definite integral)

Here are examples:

https://www.youtube.com/watch?v=rCWOdfQ3cwQ

https://www.youtube.com/watch?v=dR0pL32Zymk

Example 1 of finding a definite integral (polynomial and power rule)

Example 2 of finding a definite integral (log and IBP)

Examples 3 and 4 (two more IBP)

Riemann Sum and the Definite Integral

Further understanding of this construct of area under the curve and the definite integral is found in the notion of the Riemann sum:

Riemann sum to approx area under curve on interval [a, b]

Definition of definite integral via Riemann Sum

Improper integrals

Improper integrals basic idea

Improper integrals continues

Applications of the definite integral

Finding area between two curves

Average value of a function 1

Average value of a function 2

Position, velocity and acceleration

Finally, Present and future value of a continuous income stream

It's not the clearest video in terms of production quality, but it's really the best I've seen in terms of a spare explanation. And, the lecturer does both present and future in one video.

NOTE: Most video lecturers do not bring out the constant e^rT, as we do. But it is easier to deal with the constant term on the outside when you integrate e^(-rt) rather than in the integrand as e^(rT - rt).

Notation equivalence between our text and this video: Our time T = video M. Our f(t) = video S(t).

The easy way to remember which formula gets the multiplier is to note that, since, PV < FV, present value lacks the multiplier that future value has. No need to overthink that one.

What is the usefulness of Present Value? It's a tool for comparing what you would make if you were to hold on to your business vs if you sell it right now and invest the money you are given.