

Math 330 - 01 Homework (Spring 2022)

- LaTeX-ed solutions are encouraged and appreciated.
- If you use LaTeX, hand-in a printed version of your homework.
- You are encouraged to discuss homework problems with classmates, but such discussions should NOT include the exchange of any written material.
- Writing of homework problems should be done on an individual basis.
- References to results from the textbook and/or class notes should be included.
- The following lists should be considered partial and tentative lists until the word complete appears next to it.
- Use 8.5in x 11in paper with smooth borders. Write your **name** on top of **each page**. Staple all pages.

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\newcommand{\aut}{\textrm{Aut}} \newcommand{\sub}{\textrm{Sub}} \newcommand{\join}{\vee}
\newcommand{\bigjoin}{\bigvee} \newcommand{\meet}{\wedge} \newcommand{\bigmeet}{\bigwedge}
\newcommand{\normaleq}{\unlhd} \newcommand{\normal}{\lhd} \newcommand{\union}{\cup}
\newcommand{\intersection}{\cap} \newcommand{\bigunion}{\bigcup}
\newcommand{\bigintersection}{\bigcap} \newcommand{\sq}[2][\ ]{\sqrt{\#1}{\#2\,}}
\newcommand{\pbr}[1]{\langle \#1\rangle} \newcommand{\ds}{\displaystyle} \newcommand{\C}{\mathbb{C}}
\newcommand{\R}{\mathbb{R}} \newcommand{\Q}{\mathbb{Q}} \newcommand{\Z}{\mathbb{Z}}
\newcommand{\N}{\mathbb{N}} \newcommand{\A}{\mathbb{A}} \newcommand{\F}{\mathbb{F}}
\newcommand{\T}{\mathbb{T}} \newcommand{\ol}[1]{\overline{\#1}} \newcommand{\imp}{\Rightarrow}
\newcommand{\rimp}{\Leftarrow} \newcommand{\pinfty}{1/p^{\infty}} \newcommand{\power}{\mathcal{P}}
\newcommand{\calL}{\mathcal{L}} \newcommand{\calC}{\mathcal{C}} \newcommand{\calN}{\mathcal{N}}
\newcommand{\calB}{\mathcal{B}} \newcommand{\calF}{\mathcal{F}} \newcommand{\calR}{\mathcal{R}}
\newcommand{\calS}{\mathcal{S}} \newcommand{\calU}{\mathcal{U}} \newcommand{\calT}{\mathcal{T}}
\newcommand{\gal}{\textrm{Gal}} \newcommand{\isom}{\approx} \newcommand{\glb}{\textrm{glb}}

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Problem Set 13 (complete) Due: 05/09/2022

1. Let $f:A \rightarrow B$ and $g:C \rightarrow D$ be functions. Define $f \times g: A \times C \rightarrow B \times D$ by $(f \times g)(a,c) = (f(a), g(c))$.
Prove that if f and g are surjective, then so is $f \times g$.
2. Prove that the function $f: \mathbb{Z} \rightarrow \mathbb{N}$ given by $f(m) = \begin{cases} 2m & \text{if } m > 0, \\ -2m+1 & \text{if } m \leq 0, \end{cases}$ is bijective.
3. Prove that if A and B are finite sets, then so is $A \cup B$. Moreover, if A and B are disjoint, then $|A \cup B| = |A| + |B|$.
4. Prove Theorem 13.28. Hint: consider the function $\tan(x)$ from calculus.

Problem Set 12 (complete) Due: 05/02/2022. Board presentation: 05/06/2022

1. Prove the converse of Prop 11.2
2. Prove that for all $x, y, z, w \in \mathbb{R}$ with $z \neq 0 \neq w$,

$$\frac{x}{z} + \frac{y}{w} = \frac{xw + yz}{zw} \iff \frac{x}{z} \frac{y}{w} = \frac{xy}{zw}$$
3. Consider the set $A = \{x \in \mathbb{Q} \mid x^2 < 2\}$. Show that A is non-empty and has an upper bound in $\mathbb{Q}$, but does not have a least upper bound in $\mathbb{Q}$. Hint: by way of contradiction, assume A has a least upper bound

u in Q , and compare it with $\sqrt{2}$.

4. Prove Prop. 11.21.iii

Problem Set 11 (complete) Due: 04/19/2022. Board presentation: 04/22/2022

1. Prove part (iv) of lemma stated in class:
for $x \in \mathbb{R}$ and $r \in \mathbb{R}^+$,
(iv) $|x| \leq r$ iff $x \leq r$ and $-x \leq r$.
(Hint: use part (iii) of the same lemma.
2. Prove Prop. 10.10.iii (Hint: use 10.8.iv)
3. Prove Prop. 10.13.ii
4. Prove Prop. 10.17 (Hint: use induction)

Problem Set 10 (complete) Due: 04/11/2022. Board presentation: 04/15/2022

1. Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be functions.
 - I. Prove Prop. 9.7.ii
 - II. Prove that if $g \circ f$ is surjective, then g is surjective.
2. Prove Prop. 9.10.ii
3. Prove Prop. 9.15 (Hint: induction)
4. Prove Prop. 9.18

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