

TeX code compiled with `\documentclass{beamer}` using the Amsterdam theme.

There is one png image needed to compile slides:

problem2.png

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\begin{document} \begin{frame} \begin{block}{Steps to Solve a Related Rate} \begin{itemize} \item[\bf (i)] What
is variable? \item[\bf (ii)] Which rates are known? \item[\bf (iii)] What equation relates the variables in (ii)?
\item[\bf (iv)] Use Implicit Differentiation on the equation in (iii) to relate the rates. \end{itemize} \end{block}
If  $V$  is the volume of a cube with edge lengths  $x$  and the cube expands as time passes, find  $\frac{dV}{dt}$  in terms of  $\frac{dx}{dt}$ .
\begin{itemize} \item[\bf a)] What is  $\frac{dV}{dx}$  when  $x=4$  inches and is growing at a rate of  $2$  inches per minute?
\item[\bf b)] What is  $x$  if the volume is shrinking at  $3$  cubic inches per minute and the side length is shrinking at  $4$  inches per minute?
\item[\bf c)] Can a cube have a shrinking volume and a growing sides? \end{itemize} \end{frame} \begin{frame} \large A spherical weather balloon is being inflated at a rate of  $(0.5 \text{ m}^3/\text{sec})$ .
\begin{enumerate}[a)] \item How fast is the diameter increasing at the instant the diameter is 2 meters? \item How fast is the volume changing at that same instant? \item How fast is the surface area changing at that same instant? \end{enumerate} \end{frame} \begin{frame} \vskip 10pt As gravel is being poured into a conical pile, its volume  $V$  changes with time. As a result, the height  $h$  and radius  $r$  also change with time. Knowing that at any moment  $V = \frac{1}{3}\pi r^2 h$ , the relationship between the changes with respect to time in the volume, radius and height is
\begin{enumerate} \item  $\frac{dV}{dt} = \frac{1}{3}\pi \left( 2r\frac{dr}{dt} h + r^2\frac{dh}{dt} \right)$  \item  $\frac{dV}{dt} = \frac{1}{3}\pi \left( 2r\frac{dr}{dt} \cdot \frac{dh}{dt} \right)$  \item  $\frac{dV}{dr} = \frac{1}{3}\pi \left( 2rh + r^2\frac{dh}{dt} \right)$  \item  $\frac{dV}{dh} = \frac{1}{3}\pi \left( r^2 \right)$  \end{enumerate} \end{frame} \begin{frame} \large Imagine the following magic triangle. Its base is on a horizontal surface and no matter what you do to its height, the triangle always has area  $10 \text{ cm}^2$ . \vskip 20pt \large If you push down on the top of the triangle so that it becomes shorter at a rate of  $3 \text{ cm/sec}$ , how fast will the length of the base be changing when the triangle is  $5 \text{ cm}$  tall? \end{frame} \begin{frame} \large My neighbors have a very loud stereo. The volume knob turns half a circle (angles  $\theta$  between  $0^\circ$  and  $180^\circ$ ) and the volume of the music is given by the function  $V(\theta) = 110 \sin(\theta/2)$  decibels (dB). \vskip 20pt One night at  $3:30$  in the morning I notice an increase from a volume of  $88 \text{ dB}$  at a rate of  $1 \text{ decibel per second}$ ! At what rate can I deduce that my neighbor is turning his volume knob? \end{frame} \begin{frame} %%%%%%%%% Students should discover errors in this "solution" \small Water is leaking out of a tank shaped like a right circular cone with height  $5 \text{ m}$  and top radius  $3 \text{ m}$ . When the water level in the cone is  $2 \text{ m}$ , the water level is decreasing at a rate of  $0.1 \text{ m/s}$ . How fast is the water leaking out of the cone? \pause \begin{columns} \begin{column}{0.45\textwidth} \begin{center} \includegraphics[width=5cm]{problem2.png} \end{center} \end{column} \begin{column}{0.55\textwidth} \small The volume of the water in the cone is  $V = \frac{1}{3}\pi r^2 h$  and using the figure above and similar triangles  $\frac{r}{h} = \frac{3}{5}$ , which means  $r = \frac{3}{5}h = \frac{3}{5} \cdot 2 = \frac{6}{5}$ . This means that  $V = \frac{1}{3}\pi \left( \frac{6}{5} \right)^2 h = \frac{12\pi}{25}h$ . Taking the derivative with respect to time  $\frac{dV}{dt} = \frac{12\pi}{25}\frac{dh}{dt} = \frac{12\pi}{25} \cdot 1 = \frac{6\pi}{125} \text{ m}^3/\text{s}$ . \end{column} \end{columns} \begin{frame} \small Water is leaking out of a tank shaped like a right circular cone with height  $5 \text{ m}$  and top radius  $3 \text{ m}$ . When the water level in the cone is  $2 \text{ m}$ , the water level is decreasing at a rate of  $0.1 \text{ m/s}$ . How fast is the water leaking out of the cone? \begin{columns} \begin{column}{0.45\textwidth} \begin{center} \includegraphics[width=5cm]{problem2.png} \end{center} \end{column} \begin{column}{0.55\textwidth} \small The volume of the water in the cone is  $V = \frac{1}{3}\pi r^2 h$  and using the figure above and similar triangles  $\frac{r}{h} = \frac{3}{5}$ , which means  $r = \frac{3}{5}h$ . This means that  $V = \frac{1}{3}\pi \left( \frac{3}{5}h \right)^2 h = \frac{3\pi}{25}h^3$ . Taking the derivative with respect to time  $\frac{dV}{dt} = \frac{3\pi}{25} \cdot 3h^2 \frac{dh}{dt} = \frac{3\pi}{25} \cdot 3(2)^2 \cdot (-0.1)$ 

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$= \frac{-36\pi}{250} \frac{m^3}{s}$

A streetlight hangs 5 meters above the ground. Regina, who is 1.5 meters tall, walks away from the point under the light at a rate of 2 meters per second. How fast is her shadow lengthening when she is 7 meters away from the point under the light? (Hint: Use similar triangles.)

Suppose Regina has the ability to magically shrink herself. At what rate must she do this to keep her shadow a constant length of 3 meters? Write this as a function of only her distance from the point under the light.

A revolving beacon from a light house shines on the straight shore, and the closest point on the shore is a pier one half mile from the lighthouse. Let θ denote the angle between the lighthouse, pier, and point on the shore where the light shines.

Write the distance from the pier to the point of light as a function of θ .

What is the rate of change of the distance from the pier to the point of light with respect to θ .

Suppose θ is a function of time t . Give an expression for the rate of change of distance with respect to time t .

Suppose that the light makes 1 revolution per minute. How fast is the light traveling along the straight beach at the instant it passes over a shorepoint 1 mile away from the shorepoint nearest the searchlight?

Given that a spherical raindrop evaporates at a rate proportional to its surface area, how fast does the radius shrink?

The minute hand on a watch is 8 mm long and the hour hand is 4 mm long. How fast is the distance between the tips of the hands changing at one o'clock?

Find the shortest line segment with endpoints on the x and y axes going through the point $(1, 8)$.

What is the area of the triangle formed by the shortest line segment?

What is the rate of change of area with respect to the x -coordinate of the point on the x -axis?

For which x is the area increasing?

The speed limit on a stretch of highway is 55 mph. Highway patrol officer, Sgt. Miguel, stations himself at a point, out of view of the motorists, 50 feet off the highway. Miguel is equipped with a radar gun which measures the speed at which a car approaches his position.

He takes a reading of suspected speeders by pointing his radar gun at a point on the highway 120 feet from the point on the highway closest to him. The radar gun picks up a reading of 48 feet/sec for a green Chevy driven by Alyssa. How fast is she traveling? Is Alyssa speeding?

At a certain moment, ship A is 6 miles south and 8 miles west of ship B . Ship A at that moment is steaming east at 12 mph, while ship B is steaming north at 15 mph.

Are the ships approaching each other or separating from each other? At what rate?

Particle A moves along the positive horizontal axis, and particle B along the graph of $f(x) = -\sqrt{3x}$, $x \leq 0$. At a certain time, A is at the point $(5, 0)$ and moving with speed 3 units/sec; and B is at a distance of 3 units from the origin moving with speed 4 units/sec. At what rate is the distance between A and B changing?

From: <http://www2.math.binghamton.edu/> - **Binghamton University Department of Mathematical Sciences**

Permanent link: http://www2.math.binghamton.edu/p/calculus/resources/calculus_flipped_resources/derivatives/2.8_related_rates_text

Last update: 2015/08/29 03:35

