2021 Zassenhaus Group Theory and Friends Conference
Online conference
May 28–29 and June 4–5, 2021

PROGRAM & ABSTRACTS

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2021 Zassenhaus Group Theory and Friends Conference
Online conference
May 28–29 and June 4–5, 2021

Conference Program
Friday, May 28, Morning Session

TIME (EDT)

8:40–9:10 AM  Michael Barry (Allegheny College)
A new algorithm for decomposing modular tensor products

9:15–9:45 AM  Carmine Monetta (University of Salerno)
Series of a construction related to the
non-abelian tensor square of groups

9:50–10:20 AM Vicent Pérez Calabuig (Universitat de València)
On the computability of the abelian kernel
of an inverse semigroup

10:25–10:55 AM Juan Martínez Madrid (Universitat de València)
On the order of products of elements in finite groups

Friday May 28, Midday Session

TIME (EDT)

12:00–12:30 PM Gareth Jones (University of Southampton)
Permutation groups of prime degree: still an open problem

12:35–1:05 PM Anthony Evans (Wright State University)
The existence problem for strong complete mappings
of finite groups

1:10–1:40 PM Wil Cocke (University of Augusta)
Some thoughts on subnormal depth (part 1)

1:45–2:15 PM Ryan McCulloch (Elmira College)
Chains of normalizers of subnormal subgroups (part 2)
Friday May 28, Afternoon Session

TIME (EDT)

3:20–3:50 PM  Kenneth Johnson (Penn State University)

q-circulants

3:55-4:25 PM  Bret Benesh (College of Saint Benedict and Saint John’s University)

A game on $Z_n$

4:30–5:00 PM  Jörg Feldvoss (University of South Alabama)

Semi-simple Leibniz algebra

5:05–5:35 PM  Dustin Story (Colorado State University)

Determining synchronization of certain classes
of primitive groups of affine type
Saturday, May 29, Morning Session

TIME (EDT)

8:40–9:10 AM  Adnan Tercan (Hacettepe University)
Extending modules and related concepts

9:15–9:45 AM  Ramazan Yasar (Hacettepe University)
Weak versions of extending modules

9:50–10:20 AM Viji Thomas (Indian Institute of Science Education and Research Thiruvananthapuram)
On a theorem of Schur

10:25–10:55 AM Patrizia Longobardi (Università di Salerno)
A new criterion for solvability of a finite group

Saturday May 29, Midday Session

TIME (EDT)

12:00–12:30 PM Maria Ferrara (Università degli Studi della Campania “Luigii Vanvitelli”)
Groups whose proper subgroups are either abelian or pronormal

12:35–1:05 PM Patali Komma
Non-inner automorphisms of order $p$ in 2-generator finite $p$-groups

1:10–1:40 PM Ekaterina Kompantseva (Financial University and Moscow Pedagogical State University)
Absolute ideals of torsion-free groups of finite rank

1:45–2:15 PM Shawn Burkett (Kent State University)
Partial GVZ-groups
### Saturday May 29, Afternoon Session

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Friday, June 4, Morning Session

TIME (EDT)

8:40–9:10 AM    Xueyu Pan (Monash University)
Groups of small order type

9:15–9:45 AM    Ana Martinez-Pastor (Universitat Politècnica de València)
On products of $\pi$-decomposable groups and Hall-like theorems

9:50–10:20 AM   Carmen Melchor (Universitat de València)
Products of classes which are a union of two classes

10:25–10:55 AM  Santiago Barrera Acevedo (Monash University)
Cocyclic Hadamard matrices of order $4p$

Friday June 4, Midday Session

TIME (EDT)

12:00–12:30 PM  Agota Figula (University of Debrecen)
Stenier loops of affine type

12:35–1:05 PM    Carl-Fredrik Nyberg Brodda (University of East Anglia)
The Muller-Schupp theorem for special monoids

1:10–1:40 PM     Alexandre Grishkov (IME-USP)
Group of right multiplications of free Bol loop of exponent two

1:45–2:15 PM     Jonathan Doane (Binghamton University)
Affine duality, indeed
Friday June 4, Afternoon Session

TIME (EDT)

3:20–3:50 PM  Casey Donoven (Montana State University Northern)
3/2-generated semigroups

3:55–4:25 PM  Mark Lewis (Kent State University)
Graphs associated with groups

4:30–5:00 PM  Heiko Dietrich (Monash University)
Quotient algorithms (a.k.a. how to compute with finitely presented groups)

5:05–5:35 PM  Alexander Hulpke (Colorado State University)
Rewriting systems and group extensions
Saturday, June 5, Morning Session

TIME (EDT)

8:40–9:10 AM  Noelia Rizo (University of the Basque Country)
Blocks with few irreducible characters

9:15–9:45 AM  Lucia Sanus (Universitat de València)
Character degrees in separable groups

9:50–10:20 AM  J. Miquel Martínez (University of Valencia)
Degrees of characters in the principal block

10:25–10:55 AM  Lucas Gagnon (University of Colorado Boulder)
A gluing lemma for supercharacter theories

Saturday June 5, Midday Session

TIME (EDT)

12:35–1:05 PM  Chimere Stanley Anabanti (University of Pretoria, South Africa)
Groups with a given number of nonpower subgroups

1:10–1:40 PM  Shuchen Mu (Binghamton University)
Hochschild transfer is group transfer

1:45–2:15 PM  Matthias Neumann-Brosig (TU Braunschweig and IAV GmbH)
Computing the Frattini subgroup of a polycyclic group
Saturday June 5, Afternoon Session

TIME (EDT)

3:20–3:50 PM  Jeffrey Riedl (University of Akron)
               Congruences involving binomial coefficients arising from $p$-groups

3:55–4:25 PM  Eric Swartz (William & Mary)
               Restrictions on parameters of partial difference sets
               in nonabelian groups

4:30–5:00 PM  Ben Fine (Fairfield University)
               Elementary and universal theory of group rings
Abstracts

In order of presentation
A new algorithm for decomposing modular tensor products
Michael Barry, Allegheny College (emeritus)

Let $p$ be a prime and let $J_r$ denote a full $r \times r$ Jordan block matrix with eigenvalue 1 over a field $F$ of characteristic $p$. For positive integers $r$ and $s$ with $r \leq s$, the Jordan canonical form of the $rs \times rs$ matrix $J_r \otimes J_s$ has the form $J_{\lambda_1} \oplus J_{\lambda_2} \oplus \cdots \oplus J_{\lambda_r}$ where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r > 0$. This decomposition determines a partition $\lambda(r, s, p) = (\lambda_1, \lambda_2, \ldots, \lambda_r)$ of $rs$ but the values of the parts depend on $r$, $s$, and $p$. Let $n_1, \ldots, n_k$ be the multiplicities of the distinct parts of $\lambda(r, s, p)$ and set $c(r, s, p) = (n_1, \ldots, n_k)$. Then $c(r, s, p)$ is a composition of $r$. We present a new bottom-up algorithm for computing $c(r, s, p)$ and $\lambda(r, s, p)$ directly from the base-$p$ expansions of $r$ and $s$.

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Series of a construction related to the non-abelian tensor square
of groups

CARMINE MONETTA, University of Salerno

The non-abelian tensor square $G \otimes G$ of a group $G$, as introduced by
Brown and Loday following Miller and Dennis, is defined to be the group
generated by all symbols $g \otimes h$, with $g, h \in G$, subject to the relations

$$gg_1 \otimes h = (g^{g_1} \otimes h^{g_1})(g \otimes h) \quad \text{and} \quad g \otimes hh_1 = (g \otimes h_1)(g^{h_1} \otimes h^{h_1})$$

for all $g, g_1, h, h_1 \in G$. Despite the easy presentation, it could be difficult
to understand what group $G \otimes G$ is. Therefore one may deal with
$G \otimes G$ studying related constructions, as done by Rocco, who considered the
following operator $\nu$ in the class of groups. Let $G$ be a group and let $G^\varphi = \{g^\varphi \mid g \in G\}$ be an isomorphic copy of $G$ via $\varphi$. Then define the group

$$\nu(G) := \langle G \cup G^\varphi \mid [g_1, g_2^\varphi]^{g_3} = [g_1^{g_3}, (g_2^{g_3})^{\varphi}] = [g_1, g_2^\varphi]^{g_3}, g_1, g_2, g_3 \in G \rangle.$$

The motivation to study the group $\nu(G)$ is the commutator connection:
indeed the subgroup $[G, G^\varphi]$ of $\nu(G)$ is isomorphic to $G \otimes G$. Hence one can
investigate the structure of $\nu(G)'$ applying commutator calculus to grasp
information about $G \otimes G$. In this talk we will present new results concerning
the commutator structure of $\nu(G)$. In particular, we will show that the
derived subgroup $\nu(G)'$ is a central product of three copies of the non-
abelian tensor square $G \otimes G$, which will give in turn a new description for
the derived and lower central series of the group $\nu(G)$. Our proofs involve
both commutator calculus and techniques concerning universal properties of
the non-abelian tensor square of groups.

Joint work with Raimundo Bastos, Ricardo De Oliveira, and Noraí Rocco.

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On the computability of the abelian kernel of an inverse semigroup
Vicent Pérez Calabuig, Universitat de València

The problem of computing the abelian kernel of a finite semigroup was first solved by Delgado describing an algorithm which decides whether a given element of a finite semigroup $S$ belongs to the abelian kernel. Steinberg extended the result for any variety of abelian groups with decidable membership. In this presentation, we use a completely different approach to complete these results by giving an exact description of the abelian kernel of an inverse semigroup. An abelian group which provides this abelian kernel is also constructed. As a consequence, the proabelian closure of a finitely generated subgroup of a free group is provided by means of our computation of the abelian kernel. We are working on extending this study to obtain results on the computability of generalised kernels of inverse semigroups, such as the metabelian kernel or the $\mathcal{F}$-kernel, for extension-closed varieties of groups $\mathcal{F}$.

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On the order of products of elements in finite groups
Juan Martínez Madrid, Universitat de València

It was proved by B. Baumslag and J. Wiegold that a finite group $G$ is nilpotent if and only if $o(x)o(y) = o(xy)$ for every pair of elements $x,y$ of coprime order. In this talk, we will present several theorems that generalize this result.

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Permutation groups of prime degree: still an open problem
GARETH JONES, University of Southampton

The classification of permutation groups of prime degree is one of the oldest problems in Group Theory, going back to Lagrange and to Galois, who classified the solvable groups. A nonsolvable group of prime degree must be almost simple, and the classification of finite simple groups shows that there are just three families: alternating and symmetric groups, three sporadic examples of degree 11 and 23, and subgroups of \( PGL_n(q) \) containing \( PSL_n(q) \). However, it is an open problem whether the natural degree \( (q^n - 1)/(q - 1) \) of the latter groups is prime in finitely or infinitely many cases. (These include the Fermat and Mersenne primes for \( n = 2 \) and \( q = 2 \).) Alexander Zvonkin and I have made heuristic estimates for the distribution of such primes, based on the Bateman–Horn Conjecture from Number Theory; these are closely supported by extensive computer searches, to give strong evidence that there are infinitely many such primes, even for each fixed prime \( n \geq 3 \) and with \( q \) restricted to prime values.

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The existence problem for strong complete mappings of finite groups
ANTHONY EVANS, Wright State University

The Cayley table (multiplication table) \( M \) and the normal multiplication table \( N \) of a finite group \( G \) are latin squares. There exists a latin square orthogonal to both \( M \) and \( N \) if and only if \( G \) admits strong complete mappings. We will discuss work done on the existence problem for strong complete mappings of finite groups.

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Some thoughts on subnormal depth (part 1)
Wil Cocke, University of Augusta

As relations, subnormality is the transitive closure of normality. For a subnormal subgroup of a finite group we show that there are subnormal series for which the normalizers of each subgroup in the series form a chain. This allows us to formulate a bound on the subnormal depth of a subgroup using the index of its normalizers in the original group. Joint work with I. Marty Isaacs and Ryan McCulloch

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Chains of normalizers of subnormal subgroups (part 2)
Ryan McCulloch, Elmira College

Given a subnormal subgroup $H$ of $G$, there is a standard subnormal series for $H$ in $G$ which is obtained by repeatedly taking normal closures. It can be shown that the length of the standard subnormal series for $H$ in $G$ is as small as possible. There is another way to define a subnormal series for a subnormal subgroup $H$ of $G$. One can form a subnormal series by taking the unique largest subnormal subgroup contained in successive normalizers. We call this the greedy series for a subnormal subgroup $H$ of $G$. We show that one can build a group $G$ with a subnormal subgroup $H$ of $G$ so that the length of the greedy series for $H$ in $G$ is as large as we like, while the length of the standard subnormal series for $H$ in $G$ is small. Our construction uses the wreath product.

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The theory of determinants was a well-worked area in the 19th Century, and according to at least one modern textbook “determinants ... are of much less importance now than they once were”. This is contradicted by a body of work initiated by Gelfand and collaborators on non-commutative determinants, more precisely “quasideterminants". One of my interests is the group determinants of finite groups. These are determinants of special matrices with symmetry. For a group $G$ with a subgroup $H$ a group matrix $XG$ can be constructed as a block matrix whose blocks are group matrices $XH$. If $H$ is nonabelian the resulting determinant can be regarded as a noncommutative determinant since the blocks do not commute, which reveals many connections including surprising new perspectives on ordinary determinants. One of my interests is the group determinants of finite groups. These are determinants of special matrices with symmetry. For a group $G$ with a subgroup $H$ a group matrix $XG$ can be constructed as a block matrix whose blocks are group matrices $XH$. If $H$ is nonabelian the resulting determinant can be regarded as a noncommutative determinant since the blocks do not commute. For an abelian group $A$ it can be ordered so that the matrix $XA$ is a circulant (these are well known and there is a book by Davis on them). In joint work with Retakh, one of the collaborators of Gelfand, circulants with noncommuting elements were investigated. The results are technical. However an interesting side path was the study of $q$-circulants in which the entries satisfy $xy = qyx$ for $q$ a root of unity. The results here are interesting in that among other properties the matrices have a very simple characteristic polynomial. I will survey some results in the general case of quasideterminants and explain the results on $q$-circulants.

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A game on $\mathbb{Z}_n$

BRET BENESH, College of Saint Benedict and Saint John’s University

We consider a pair of games where two players alternately select previously-unselected elements of $\mathbb{Z}_n$ given a particular starting element. On each turn, the player either adds or multiplies the element they selected to the result of the previous turn. In one game, the first player wins if the final result is 0; in the other, the second player wins if the final result 0. We determine which player has the winning strategy for both games except for the latter game with nonzero starting element when $n \in \{2p, 4p\}$ for some odd prime $p$. This is joint work with Robert Campbell.

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Semi-simple Leibniz algebra

JÖRG FELDVOSS, University of South Alabama

Leibniz algebras were introduced by Bloh and Loday as non-commutative analogues of Lie algebras. Many results for Lie algebras have been proven to hold for Leibniz algebras but there are also several results that are not true in this more general context. In my talk I will describe the structure of finite-dimensional semi-simple Leibniz algebras over a field of characteristic zero. They are hemi-semidirect products of a semi-simple Lie algebra and a completely reducible module over this Lie algebra without non-zero trivial submodules. If time permits, I will apply this to derive some results on the derivation algebras of such Leibniz algebras.

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Determining synchronization of certain classes of primitive groups of affine type
DUSTIN STORY, Colorado State University

The class of permutation groups includes 2-homogeneous groups, synchronizing groups, and primitive groups. Moreover, 2-homogeneous implies synchronizing, and synchronizing in turn implies primitivity. A complete classification of synchronizing groups remains an open problem. The search exists amongst the primitive groups, looking for examples of synchronizing and nonsynchronizing. Our main results are constructive proofs that show two classes of affine groups are nonsynchronizing.

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Extending modules and related concepts
ADNAN TERCAN, Hacettepe University, Department of Mathematics

Let $R$ be a ring with identity and $M$ an right $R$-module. Recall that $M$ is called a right CS (or extending or $C_1$) if every submodule of $M$ is essential in a direct summand of $M$. There are several generalizations of CS notion. In this talk first we will give some basic properties of CS modules then we deal with some of the generalizations of them.

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Weak versions of extending modules
RAMAZAN YASAR, Hacettepe University, Hacettepe-ASO 1.OSB Vocational School

Let $R$ be a ring with identity and $M$ an right $R$-module. Recall that $M$ is called a right Weak CS (or WCS) if every semisimple submodule of $M$ is essential in a direct summand of $M$. There are several generalizations of WCS notion. In this talk first we will give some well-known properties of WCS modules. Then we focus on some of the useful generalizations of them.

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On a theorem of Schur

Viji Thomas, Indian Institute of Science Education and Research
Thiruvananthapuram

We will discuss how the exponent of a central quotient of a group affects the exponent of the commutator subgroup. We will survey some results in this direction and then present our recent results. This is joint work with Komma Patali.

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A new criterion for solvability of a finite group

Patrizia Longobardi, Universitá di Salerno

The problem of detecting structural properties of a finite group by looking at some functions related to the structure of $G$ has been recently considered by various authors. For instance, H. Amiri, S.M. Jafari Amiri and I.M. Isaacs introduced the function

$$
\psi(G) = \sum_{x \in G} o(x),
$$

where $o(x)$ denotes the order of the element $x$. Many results concerning the function $\psi(G)$ have been obtained by many authors. Other functions have been introduced and studied, for instance

$$
\rho(G) = \prod_{x \in G} o(x),
$$

the product of element orders of a finite group $G$. Now we are concerned with another function related to the structure of a finite group $G$. We write

$$
\sigma_1(G) = \frac{1}{|G|} \sum_{H \leq G} |H|.
$$

Answering to a problem posed by M. Tărnăuceanu, together with Marcel Herzog and Mercede Maj, we have shown that, if $\sigma_1(G) < \frac{117}{20}$, then $G$ is solvable.

The aim of my talk is to provide a brief survey and outline this proof.

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Groups whose proper subgroups are either abelian or pronormal

Maria Ferrara, Università degli Studi della Campania “Luigii Vanvitelli”

A subgroup $X$ of a group $G$ is called pronormal in $G$ if for each $g \in G$ the subgroups $X$ and $X^g$ are conjugate in $\langle X, X^g \rangle$. Of course, normal subgroups and maximal subgroups are example of pronormal subgroups. The structure of groups in which all subgroups are pronormal is well known and here I present results on prohamiltonian groups, that is, groups in which all non-abelian subgroups are pronormal.

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Non-inner automorphisms of order $p$ in 2-generator finite $p$-groups

Patali Komma,

A long-standing conjecture asserts that every finite nonabelian $p$-group admits a non-inner automorphism of order $p$. We prove this conjecture for every 2-generator finite $p$-group, $p \geq 5$. As a consequence, we deduce that finite $p$-groups of coclass 4, and coclass 5 have a non-inner automorphism of order $p$, $p \geq 5$. We achieve this using the notion of Camina triples.

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Absolute ideals of torsion-free groups of finite rank
Ekaterina Kompantseva, Financial University under the Government of the Russian Federation; Moscow Pedagogical State University

A subgroup $A$ of an abelian group $G$ is called its absolute ideal if $A$ is an ideal of any ring on $G$. Rings on an abelian group $G$, in which there do not exist different ideals without an absolute, is called an AI-ring. If there exits an AI-ring on an abelian group, then the group is called an RAI-group. The problem of describing of RAI-groups is formulated by L. Fuchs. Obviously, every fully invariant subgroup of an abelian group $G$ is an absolute ideal of $G$. E. Fried formulated the problem of studying of abelian groups for which the converse is true, that is every absolute ideal is a fully invariant subgroup. Such groups are called “af i-groups”. The aim of this talk is to describe RAI-groups and af i-groups in the class of abelian Murley groups.

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Partial GVZ-groups
Shawn Burkett, Kent State University

Following the literature, a group $G$ is called a group of central type if $G$ has an irreducible character that vanishes on $G \setminus Z(G)$. Motivated by this definition, we say that a character $\chi \in \text{irr}G$ has central type if $\chi$ vanishes on $G \setminus Z(\chi)$, where $Z(\chi)$ is the center of $\chi$. Groups where every irreducible character has central type have been studied previously under the name GVZ-groups (and several other names) in the literature. In this talk we consider the groups $G$ that possess a nontrivial, normal subgroup $N$ such that every character of $G$ either contains $N$ in its kernel or has central type. The structure of these groups is surprisingly limited and has many aspects in common with both central type groups and GVZ-groups.

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Maximal subgroups of Thompson’s groups V
Rachel Skipper, The Ohio State University

We will use graphs to build a family of maximal subgroups of Thompson’s group $V$. This is a joint work with Jim Belk, Collin Bleak, and Martyn Quick.

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Finiteness properties of normal subgroups of the Stein group $F_{2,3}$
Matt Zaremsky, University of Albany (SUNY)

In this talk I will discuss recent joint work with Rob Spahn, in which we give a complete computation of all the Bieri-Neumann-Strebel-Renz invariants of the Stein group $F_{2,3}$. This is a relative of Thompson’s group $F$ that has proven over the years to be even more unusual than the famously unusual $F$. A consequence of our computation of the BNSR-invariants is a complete classification of the finiteness properties of every normal subgroup of $F_{2,3}$. In particular we prove that the kernel of every map from $F_{2,3}$ to $\mathbb{Z}$ is finitely presented, and even of type $F_\infty$, but there exist maps from $F_{2,3}$ to $\mathbb{Z}^2$ that are not even finitely generated. This makes the Stein group the first group with fully computed BNSR-invariants known to have this property. The talk will be self-contained, and no prior knowledge of any of the above topics will be needed.

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Closed subgroups of the R. Thompson group $F$
Mark Sapir, Vanderbilt University

Joint with Gili Golan. We prove that the closure of any finitely generated subgroup of $F$ is finitely generated and undistorted in $F$.

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Laconic semigroups, laconic varieties, and the membership problem
ZORAN SUNIC, Hofstra University

Let $G = \langle A \rangle$ be a finitely generated group and $S$ a finitely generated subsemigroup of $G$, given by a finite list of group words over $A$ that generate $S$. The Membership Problem for $S$ in $G$ asks for an algorithm that, given a group word $w$ over $A$, decides if the element of $G$ represented by $w$ belongs to $S$. We recall a technique, upper distortion, introduced by Margolis, Meakin, and the speaker, that solves the Membership Problem in certain situations involving groups and subsemigroups, and we extend it beyond this limited context. (A semigroup $S$ is laconic if, for every finitely generated free semigroup $X^+$, every homomorphism $f : X^+ \to S$, and every element $s$ of $S$, the fiber $\phi^{-1}(s)$ is finite. In other words, every element of $S$ is represented by only finitely many words over $X$.)

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Groups of small order type
XUEYU (EILEEN) PAN, Monash University

For my MPhil project I investigate groups whose orders factorise into few primes. Theoretical classifications of these groups exist in the literature, but it is often difficult to extract the results. The aim of my thesis is to describe these classifications in a unified, modern language. An important and practical aspect of our new descriptions is that they lead to new efficient construction and determination algorithms for groups of some small order types.

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On products of $\pi$-decomposable groups and Hall-like theorems

Ana Martinez-Pastor, Universitat Politècnica de València

Let $\pi$ a set of odd primes and let $G = AB$ a finite group which is the product of two $\pi$-decomposable subgroups $A = A_\pi \times A_{\pi'}$ and $B = B_\pi \times B_{\pi'}$. We discuss in this talk some Hall-like theorems for such a factorized group $G$ and, in particular, we show that $G$ has a unique conjugacy class of Hall $\pi$-subgroups, and any $\pi$-subgroup is contained in a Hall $\pi$-subgroup (i.e. $G$ satisfies property $D_\pi$).

(Joint work with Lev S. Kazarin and M. Dolores Pérez-Ramos.)

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Products of classes which are a union of two classes

Carmen Melchor, Universitat de València

There are several results about non-simplicity, solvability and normal structure of finite groups related to the product of conjugacy classes. In this framework it is well-known the Arad-Herzog’s conjecture which asserts that a finite group having two classes such that its product is again a conjugacy class is not a non-abelian simple group. Now, we fix our focus on the case when the product of two classes is the union of two conjugacy classes.

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Cocyclic Hadamard matrices were introduced by de Launey and Horadam as a class of Hadamard matrices with interesting algebraic properties. Catháin and Ráder described a classification algorithm for cocyclic Hadamard matrices of order $4n$ based on relative difference sets in groups of order $8n$; this led to the classification of cocyclic Hadamard matrices of order at most 36. Based on work of de Launey and Flannery, we investigated in detail the structure of cocyclic Hadamard matrices of order $4p$, with $p$ prime. This led to a classification algorithm and the determination of cocyclic Hadamard matrices of order 44 and 52 up to equivalence.

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Steiner loops of affine type
AGOTA FIGULA, University of Debrecen, Institute of Mathematics

Steiner loops of affine type are associated to arbitrary Steiner triple systems. They behave to elementary abelian 3-groups as arbitrary Steiner Triple Systems behave to affine geometries over GF(3). We investigate algebraic and geometric properties of these loops often in connection to configurations. Steiner loops of affine type, as extensions of normal subloops by factor loops, are studied. We prove that the multiplication group of every Steiner loop of affine type with $n$ elements is contained in the alternating group $A_n$ and we give conditions for those loops having $A_n$ as their multiplication groups (and hence for the loops being simple).

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The Muller-Schupp theorem for special monoids
Carl-Fredrik Nyberg Brodda, University of East Anglia

The study of ‘special’ monoids goes back to S. I. Adian and his student G. S. Makanin, who reduced the word problem of such monoids to the same problem for their group of units. There are many ways in which special monoids behave like “generalised” groups, and studying them has many similarities with combinatorial group theory. I will give a brief overview of the main ideas involved in studying special monoids – including the definition! – and the role played by the group of units. I will then present some recent full generalisations of the celebrated Muller-Schupp theorem from virtually free groups to special monoids with virtually free groups of units.

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Group of right multiplications of free Bol loop of exponent two
Alexandre Grishkov, IME-USP

A set $B$ with multiplication $a \cdot b$ is a (right) Bol loop of exponent two if for every $a, b \in B$ we have $a \cdot ((b \cdot c) \cdot b) = ((a \cdot b) \cdot c) \cdot b$, $a \cdot a = e$, where $e$ is the unit of $B$. By definition a group Multr($B$) is a subgroup of the group Sym($B$) of all permutations of the set $B$, generated by Let $B_n$ be a free Bol loop of exponent two with $n$ free generators. We proved that Multr($B_n$) is a free product of cyclic groups of order two. Moreover, we construct canonical (in some sense) subset $T$ of $B$. This result was obtained in collaboration with M. Rasskazova (Russia) and G. Souza dos Anjos (Brazil).

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**Affine duality, indeed**

**JONATHAN DOANE**, Binghamton University

Rings with unity and bounded lattices have two underlying monoid structures - one with 0 and another with 1. There are exactly four “twice monoid” structures for which \{0, 1\} can be equipped; two of these actually resemble Boolean rings (BRs), one is a bounded distributive lattice (BDL), and the last is an elementary Abelian 2-group with 1, called A. Since BRs and BDLs are dualizable to Boolean spaces and Priestley spaces, respectively, it is natural to ask whether or not A is dualizable as well. The aim of this talk is to answer this question in the affirmative.

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**3/2-generated semigroups**

**CASEY DONOVEN**, Montana State University Northern

A 3/2-generated group is a group in which every nontrivial element belongs to a two-element generating set (hence, slightly stronger than being 2-generated). It is conjectured that finite group is 3/2-generated if and only if all of its proper quotients are cyclic. In this talk, I will define 3/2-generated semigroups, and present a complete classification of finite 3/2-generated semigroups.

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**Graphs associated with groups**

**MARK LEWIS**, Kent State University

There are many different graphs that have been associated with groups. In our talk, we consider the commuting graph and several related graphs. In particular, we will survey the known results regarding these graphs. We will present a new result that extends to Parker’s result on the commuting graphs of centerless solvable groups. We will present some new results on the cyclic graph (a.k.a. the enhanced power graph).

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Quotient algorithms (a.k.a. how to compute with finitely presented groups)
Heiko Dietrich, Monash University

I will survey some of the famous quotient algorithms that can be used to compute efficiently with finitely presented groups. The last part of the talk will be about joint work with Alexander Hulpke (Colorado State University) on quotient algorithms for non-solvable groups.

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Rewriting systems and group extensions
Alexander Hulpke, Colorado State University

I will show how the confluence condition of rewriting systems can be used to calculate 2-cohomology groups and to construct group extensions. This will be illustrated by examples in GAP. This is based on joint work with Heiko Dietrich (Melbourne)

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Blocks with few irreducible characters
Noelia Rizo, University of the Basque Country

Let $G$ be a finite group, let $p$ be a prime number and let $B$ be a $p$-block of $G$ with defect group $D$. Studying the structure of $D$ by means of the knowledge of some aspects of $B$ is a main area in character theory of finite groups. Let $k(B)$ be the number of irreducible characters in the $p$-block $B$. It is well-known that $k(B) = 1$ if, and only if, $D$ is trivial. It is also true that $k(B) = 2$ if, and only if, $|D| = 2$. For blocks $B$ with $k(B) = 3$ it is conjectured that $|D| = 3$. When $B$ is the principal block, more can be said. For instance, the structure of defect groups of principal blocks having $3$ and $4$ irreducible characters is known. In this work we go one step further and analyze the structure of $D$ when $k(B) = 5$. This is a joint work with Mandi Schaeffer Fry and Carolina Vallejo.

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Character degrees in separable groups
LUCIA SANUS, Universitat de València

Under some separability conditions, many results on character degrees of finite groups can be unified in a single statement. We present a characterization, under some separability conditions, of when \( \text{Irr}_\pi(G) = \text{Irr}_\rho(G) \) where \( \pi \) and \( \rho \) are sets of primes and \( \text{Irr}_\pi(G) \) is the set of irreducible characters \( \chi \) of \( G \) such that all the primes dividing \( \chi(1) \) lie in \( \pi \). This generalizes the well-known theorems of Ito-Michler and Thompson on character degrees. This is joint work with G. Navarro and N. Rizo.

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Degrees of characters in the principal block
J. MIQUEL MARTÍNEZ, University of Valencia

Let \( G \) be a finite group and let \( p \) be a prime. The set of complex irreducible characters in the principal \( p \)-block of \( G \) is rich enough that their degrees encode information of the structure of the group \( G \). We study the case where the set of degrees of characters in the principal \( p \)-block of \( G \) has size at most 2, finding information about the structure of \( G \) and its Sylow \( p \)-subgroups. We will also show some related results on similar problems for arbitrary \( p \)-blocks.

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A gluing lemma for supercharacter theories
LUCAS GAGNON, University of Colorado Boulder

Supercharacter theory is a tool for approximating the representation theory of “difficult” groups, like the Sylow $p$-subgroups of the finite general linear groups. It seems likely that most groups have many supercharacter theories, which emphasize and suppress different interesting representation-theoretic properties, but there are few known constructions which work for an arbitrary group. In this talk I will describe a new approach to constructing supercharacter theories for an arbitrary finite group via a lattice of normal subgroups and supercharacter-theoretic data from some subintervals of this lattice, similar to the gluing lemma. This construction allows for the construction of many different supercharacter theories when the group has an abundance of normal subgroups (e.g. $p$-groups). When the lattice subintervals come from a Galois connection, it is particularly easy to describe the resulting supercharacter theory, and nice properties result.

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Groups with a given number of nonpower subgroups
CHIMERE STANLEY ANABANTI, University of Pretoria, South Africa

It is well-known that no group has either exactly 1 or exactly 2 nonpower subgroups. In this talk, we give a classification of finite groups that have exactly 3 nonpower subgroups, show that there is a unique finite group with exactly 4 nonpower subgroups, as well as show that for all integers $k > 4$, there are infinitely many groups with exactly $k$ nonpower subgroups. This is joint work with Sarah Hart.

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Hochschild transfer is group transfer  
SHUCHEN MU, Binghamton University

It was proved that Hochschild homology of a group ring splits into direct sum of group homology of centralizers. When given a finite index subgroup, we can define transfer map on Hochschild homology which becomes group homology transfer on corresponding summands. We will present the proof on chain complex level and the problem becomes a group theoretical one.

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Computing the Frattini subgroup of a polycyclic group  
MATTHIAS NEUMANN-BROSIG, TU Braunschweig, Germany and IAV GmbH

A first algorithm to compute the Frattini subgroup of a polycyclic-by-finite group was introduced by Baumslag, Cannonito, Segal & Robinson (1991). Here we present a new method to determine the Frattini subgroup of a polycyclic group. This new method has the advantage that it can be implemented and is practical for a variety of applications.

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Congruences involving binomial coefficients arising from $p$-groups  
JEFFREY RIEDL, University of Akron

Let $p$ be an arbitrary prime and let $n > 1$ be an integer. Let $W$ be the regular wreath product $C \wr H$ where $C$ and $H$ are cyclic groups of orders $p^2$ and $p^n$ respectively. Thus $W$ is the semidirect product $B \rtimes H$ where $B$ is the direct product of $p^n$ copies of $C$, with $H$ permuting these direct factors. Binomial coefficients arise naturally in the structure of the lower central series of $W$. Our study of $W$ has revealed some interesting congruences modulo $p^2$ involving binomial coefficients. We discuss these congruences.

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Restrictions on parameters of partial difference sets in nonabelian groups
ERIC SWARTZ, William & Mary

A partial difference set $S$ in a finite group $G$ satisfying $1 \notin S$ and $S = S^{-1}$ corresponds to an undirected strongly regular Cayley graph Cay$(G, S)$. While the case when $G$ is abelian has been thoroughly studied, there are comparatively few results when $G$ is nonabelian. We provide restrictions on the parameters of a partial difference set that apply to both abelian and nonabelian groups and are especially effective in groups with a nontrivial center. In particular, these results apply to $p$-groups, and we are able to rule out the existence of partial difference sets in many instances. This is joint work with Gabrielle Tauscheck.

eswartz@gmail.com
Joint work with Anthony Gaglione, Martin Kreuzer, Gerhard Rosenberger, and Dennis Spellman

In a series of papers the above authors examined the relationship between the universal and elementary theory of a group ring $R[G]$ and the corresponding universal and elementary theory of the associated group $G$ and ring $R$. Here we assume that $R$ is a commutative ring with identity $1 \neq 0$. Of course, these are relative to an appropriate logical language $L_0$, $L_1$, $L_2$ for groups, rings, and group rings, respectively. Axiom systems for these were provided. Kharlampovich and Myasnikov as part of the proof of the Tarski theorems prove that the elementary theory of free groups is decidable. For a group ring they have proven that the first-order theory (in the language of ring theory) is not decidable and have studied equations over group rings, especially for torsion-free hyperbolic groups. We examine and survey extensions of Tarski-like results to the collection of group rings and examine relationships between the universal and elementary theories of the corresponding groups and rings and the corresponding universal theory of the formed group ring. To accomplish this we introduce different first-order languages with equality whose model classes are respectively groups, rings, and group rings. We prove that if $R[G]$ is elementarily equivalent to $S[H]$ then simultaneously the group $G$ is elementarily equivalent to the group $H$ and the ring $R$ is elementarily equivalent to the ring $S$ with respect to the appropriate languages. Further if $G$ is universally equivalent to a nonabelian free group $F$ and $R$ is universally equivalent to the integers $\mathbb{Z}$ then $R[G]$ is universally equivalent to $\mathbb{Z}[F]$, again with respect to an appropriate language. It was proved that if $R[G]$ is elementarily equivalent to $S[H]$ with respect to $L_2$, then simultaneously the group $G$ is elementarily equivalent to the group $H$ with respect to $L_0$, and the ring $R$ is elementarily equivalent to the ring $S$ with respect to $L_1$.

The structure of group rings is related to the Kaplansky zero-divisor conjecture. A Kaplansky group is a torsion-free group which satisfies the Kaplansky conjecture. We next show that each of the classes of left-orderable groups and orderable groups is a quasivariety with undecidable theory. In the case of orderable groups, we find an explicit set of universal axioms. We have that $\mathcal{K}$ the class of Kaplansky groups is the model class of a set of universal sentences in the language of group theory. We also give a characterization of when two groups in $\mathcal{K}$ or more generally two torsion-free groups are universally equivalent.
Finally we consider $F$ to be a rank 2 free group and $\mathbb{Z}$ be the ring of integers. We call $\mathbb{Z}[F]$ a free group ring. Examining the universal theory of the free group ring $\mathbb{Z}[F]$ the hazy conjecture was made that the universal sentences true in $\mathbb{Z}[F]$ are precisely the universal sentences true in $F$ modified appropriately for group ring theory and the converse that the universal sentences true in $F$ are the universal sentences true in $\mathbb{Z}[F]$ modified appropriately for group theory. We prove that this conjecture is true in terms of axiom systems for $\mathbb{Z}[F]$.

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2021 Zassenhaus Group Theory and Friends Conference
Online conference
May 28–29 and June 4–5, 2021

Abstracts

Alphabetical by Speaker
Groups with a given number of nonpower subgroups
Chimere Stanley Anabanti, University of Pretoria, South Africa

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Cocyclic Hadamard matrices of order \( 4p \)
Santiago Barrera Acevedo, Monash University

Cocyclic Hadamard matrices were introduced by de Launey and Horadam as a class of Hadamard matrices with interesting algebraic properties. Cath\'\-A\-jin and R\'\-A\-\-der described a classification algorithm for cocyclic Hadamard matrices of order \( 4n \) based on relative difference sets in groups of order \( 8n \); this led to the classification of cocyclic Hadamard matrices of order at most 36. Based on work of de Launey and Flannery, we investigated in detail the structure of cocyclic Hadamard matrices of order \( 4p \), with \( p \) prime. This led to a classification algorithm and the determination of cocyclic Hadamard matrices of order 44 and 52 up to equivalence.

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A new algorithm for decomposing modular tensor products

MICHAEL BARRY, Allegheny College (emeritus)

Let $p$ be a prime and let $J_r$ denote a full $r \times r$ Jordan block matrix with eigenvalue 1 over a field $F$ of characteristic $p$. For positive integers $r$ and $s$ with $r \leq s$, the Jordan canonical form of the $rs \times rs$ matrix $J_r \otimes J_s$ has the form $J_{\lambda_1} \oplus J_{\lambda_2} \oplus \cdots \oplus J_{\lambda_r}$ where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r > 0$. This decomposition determines a partition $\lambda(r, s, p) = (\lambda_1, \lambda_2, \ldots, \lambda_r)$ of $rs$ but the values of the parts depend on $r$, $s$, and $p$. Let $n_1, \ldots, n_k$ be the multiplicities of the distinct parts of $\lambda(r, s, p)$ and set $c(r, s, p) = (n_1, \ldots, n_k)$. Then $c(r, s, p)$ is a composition of $r$. We present a new bottom-up algorithm for computing $c(r, s, p)$ and $\lambda(r, s, p)$ directly from the base-$p$ expansions of $r$ and $s$.

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A game on $\mathbb{Z}_n$

BRET BENESH, College of Saint Benedict and Saint John’s University

We consider a pair of games where two players alternately select previously-unselected elements of $\mathbb{Z}_n$ given a particular starting element. On each turn, the player either adds or multiplies the element they selected to the result of the previous turn. In one game, the first player wins if the final result is 0; in the other, the second player wins if the final result 0. We determine which player has the winning strategy for both games except for the latter game with nonzero starting element when $n \in \{2p, 4p\}$ for some odd prime $p$. This is joint work with Robert Campbell.

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Partial GVZ-groups
SHAWN BURKETT, Kent State University

Following the literature, a group $G$ is called a group of central type if $G$ has an irreducible character that vanishes on $G \setminus Z(G)$. Motivated by this definition, we say that a character $\chi \in \text{irr} G$ has central type if $\chi$ vanishes on $G \setminus Z(\chi)$, where $Z(\chi)$ is the center of $\chi$. Groups where every irreducible character has central type have been studied previously under the name GVZ-groups (and several other names) in the literature. In this talk we consider the groups $G$ that possess a nontrivial, normal subgroup $N$ such that every character of $G$ either contains $N$ in its kernel or has central type. The structure of these groups is surprisingly limited and has many aspects in common with both central type groups and GVZ-groups.

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Some thoughts on subnormal depth (part 1)
WIL COCKE, University of Augusta

As relations, subnormality is the transitive closure of normality. For a subnormal subgroup of a finite group we show that there are subnormal series for which the normalizers of each subgroup in the series form a chain. This allows us to formulate a bound on the subnormal depth of a subgroup using the index of its normalizers in the original group. Joint work with I. Marty Isaacs and Ryan McCulloch

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Quotient algorithms (a.k.a. how to compute with finitely presented groups)
HEIKO DIETRICH, Monash University

I will survey some of the famous quotient algorithms that can be used to compute efficiently with finitely presented groups. The last part of the talk will be about joint work with Alexander Hulpke (Colorado State University) on quotient algorithms for non-solvable groups.

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Affine duality, indeed
JONATHAN DOANE, Binghamton University

Rings with unity and bounded lattices have two underlying monoid structures - one with 0 and another with 1. There are exactly four “twice monoid” structures for which \{0, 1\} can be equipped; two of these actually resemble Boolean rings (BRs), one is a bounded distributive lattice (BDL), and the last is an elementary Abelian 2-group with 1, called \textbf{A}. Since BRs and BDLs are dualizable to Boolean spaces and Priestley spaces, respectively, it is natural to ask whether or not \textbf{A} is dualizable as well. The aim of this talk is to answer this question in the affirmative.

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3/2-generated semigroups
CASEY DONOVEN, Montana State University Northern

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The existence problem for strong complete mappings of finite groups
ANTHONY EVANS, Wright State University

The Cayley table (multiplication table) \(M\) and the normal multiplication table \(N\) of a finite group \(G\) are latin squares. There exists a latin square orthogonal to both \(M\) and \(N\) if and only if \(G\) admits strong complete mappings. We will discuss work done on the existence problem for strong complete mappings of finite groups.

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Semi-simple Leibniz algebra
JÖRG FELDVOSS, University of South Alabama

Leibniz algebras were introduced by Bloh and Loday as non-commutative analogues of Lie algebras. Many results for Lie algebras have been proven to hold for Leibniz algebras but there are also several results that are not true in this more general context. In my talk I will describe the structure of finite-dimensional semi-simple Leibniz algebras over a field of characteristic zero. They are hemi-semidirect products of a semi-simple Lie algebra and a completely reducible module over this Lie algebra without non-zero trivial submodules. If time permits, I will apply this to derive some results on the derivation algebras of such Leibniz algebras.

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Groups whose proper subgroups are either abelian or pronormal
MARIA FERRARA, Università degli Studi della Campania “Luigii Vanvitelli”

A subgroup $X$ of a group $G$ is called pronormal in $G$ if for each $g \in G$ the subgroups $X$ and $X^g$ are conjugate in $\langle X, X^g \rangle$. Of course, normal subgroups and maximal subgroups are example of pronormal subgroups. The structure of groups in which all subgroups are pronormal is well known and here I present results on prohamiltonian groups, that is, groups in which all non-abelian subgroups are pronormal.

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Steiner loops of affine type
AGOTA FIGULA, University of Debrecen, Institute of Mathematics

Steiner loops of affine type are associated to arbitrary Steiner triple systems. They behave to elementary abelian 3-groups as arbitrary Steiner Triple Systems behave to affine geometries over GF(3). We investigate algebraic and geometric properties of these loops often in connection to configurations. Steiner loops of affine type, as extensions of normal subloops by factor loops, are studied. We prove that the multiplication group of every Steiner loop of affine type with $n$ elements is contained in the alternating group $A_n$ and we give conditions for those loops having $A_n$ as their multiplication groups (and hence for the loops being simple).

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Joint work with Anthony Gaglione, Martin Kreuzer, Gerhard Rosenberger, and Dennis Spellman

In a series of papers the above authors examined the relationship between the universal and elementary theory of a group ring $R[G]$ and the corresponding universal and elementary theory of the associated group $G$ and ring $R$. Here we assume that $R$ is a commutative ring with identity $1 \neq 0$. Of course, these are relative to an appropriate logical language $L_0$, $L_1$, $L_2$ for groups, rings, and group rings, respectively. Axiom systems for these were provided. Kharlampovich and Myasnikov as part of the proof of the Tarski theorems prove that the elementary theory of free groups is decidable. For a group ring they have proven that the first-order theory (in the language of ring theory) is not decidable and have studied equations over group rings, especially for torsion-free hyperbolic groups. We examine and survey extensions of Tarski-like results to the collection of group rings and examine relationships between the universal and elementary theories of the corresponding groups and rings and the corresponding universal theory of the formed group ring. To accomplish this we introduce different first-order languages with equality whose model classes are respectively groups, rings, and group rings. We prove that if $R[G]$ is elementarily equivalent to $S[H]$ then simultaneously the group $G$ is elementarily equivalent to the group $H$ and the ring $R$ is elementarily equivalent to the ring $S$ with respect to the appropriate languages. Further if $G$ is universally equivalent to a nonabelian free group $F$ and $R$ is universally equivalent to the integers $\mathbb{Z}$ then $R[G]$ is universally equivalent to $\mathbb{Z}[F]$, again with respect to an appropriate language. It was proved that if $R[G]$ is elementarily equivalent to $S[H]$ with respect to $L_2$, then simultaneously the group $G$ is elementarily equivalent to the group $H$ with respect to $L_0$, and the ring $R$ is elementarily equivalent to the ring $S$ with respect to $L_1$.

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A gluing lemma for supercharacter theories

LUCAS GAGNON, University of Colorado Boulder

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Group of right multiplications of free Bol loop of exponent two
ALEXANDRE GRISHKOV, IME-USP

A set $B$ with multiplication $a \cdot b$ is a (right) Bol loop of exponent two if for every $a, b \in B$ we have $a \cdot ((b \cdot c) \cdot b) = ((a \cdot b) \cdot c) \cdot b$, $a \cdot a = e$, where $e$ is the unit of $B$. By definition a group Multr($B$) is a subgroup of the group Sym($B$) of all permutations of the set $B$, generated by Let $B_n$ be a free Bol loop of exponent two with $n$ free generators. We proved that Multr($B_n$) is a free product of cyclic groups of order two. Moreover, we construct canonical (in some sense) subset $T$ of $B$. This result was obtained in collaboration with M. Rasskazova (Russia) and G. Souza dos Anjos (Brazil).

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Rewriting systems and group extensions
ALEXANDER HULPKE, Colorado State University

I will show how the confluence condition of rewriting systems can be used to calculate 2-cohomology groups and to construct group extensions. This will be illustrated by examples in GAP. This is based on joint work with Heiko Dietrich (Melbourne)

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The theory of determinants was a well-worked area in the 19th Century, and according to at least one modern textbook “determinants ... are of much less importance now than they once were”. This is contradicted by a body of work initiated by Gelfand and collaborators on non-commutative determinants, more precisely “quasideterminants” One of my interests is the group determinants of finite groups. These are determinants of special matrices with symmetry. For a group $G$ with a subgroup $H$ a group matrix $XG$ can be constructed as a block matrix whose blocks are group matrices $XH$. If $H$ is nonabelian the resulting determinant can be regarded as a noncommutative determinant since the blocks do not commute, which reveals many connections including surprising new perspectives on ordinary determinants. One of my interests is the group determinants of finite groups. These are determinants of special matrices with symmetry. For a group $G$ with a subgroup $H$ a group matrix $XG$ can be constructed as a block matrix whose blocks are group matrices $XH$. If $H$ is nonabelian the resulting determinant can be regarded as a noncommutative determinant since the blocks do not commute. For an abelian group $A$ it can be ordered so that the matrix $XA$ is a circulant (these are well known and there is a book by Davis on them). In joint work with Retakh, one of the collaborators of Gelfand, circulants with noncommuting elements were investigated. The results are technical. However an interesting side path was the study of $q$-circulants in which the entries satisfy $xy = qyx$ for $q$ a root of unity. The results here are interesting in that among other properties the matrices have a very simple characteristic polynomial. I will survey some results in the general case of quasideterminants and explain the results on $q$-circulants.

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Permutation groups of prime degree: still an open problem
GARETH JONES, University of Southampton

The classification of permutation groups of prime degree is one of the oldest problems in Group Theory, going back to Lagrange and to Galois, who classified the solvable groups. A nonsolvable group of prime degree must be almost simple, and the classification of finite simple groups shows that there are just three families: alternating and symmetric groups, three sporadic examples of degree 11 and 23, and subgroups of $\text{PGL}_n(q)$ containing $\text{PSL}_n(q)$. However, it is an open problem whether the natural degree $(q^n - 1)/(q - 1)$ of the latter groups is prime in finitely or infinitely many cases. (These include the Fermat and Mersenne primes for $n = 2$ and $q = 2$.) Alexander Zvonkin and I have made heuristic estimates for the distribution of such primes, based on the Bateman–Horn Conjecture from Number Theory; these are closely supported by extensive computer searches, to give strong evidence that there are infinitely many such primes, even for each fixed prime $n \geq 3$ and with $q$ restricted to prime values.

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Non-inner automorphisms of order $p$ in 2-generator finite $p$-groups
PATALI KOMMA,

A long-standing conjecture asserts that every finite nonabelian $p$-group admits a non-inner automorphism of order $p$. We prove this conjecture for every 2-generator finite $p$-group, $p \geq 5$. As a consequence, we deduce that finite $p$-groups of coclass 4, and coclass 5 have a non-inner automorphism of order $p$, $p \geq 5$. We achieve this using the notion of Camina triples.

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Absolute ideals of torsion-free groups of finite rank
Ekaterina Kompantseva, Financial University under the Government of the Russian Federation; Moscow Pedagogical State University

A subgroup $A$ of an abelian group $G$ is called its absolute ideal if $A$ is an ideal of any ring on $G$. Rings on an abelian group $G$, in which there do not exist different ideals without an absolute, is called an AI-ring. If there exits an AI-ring on an abelian group, then the group is called an RAI-group. The problem of describing of RAI-groups is formulated by L. Fuchs. Obviously, every fully invariant subgroup of an abelian group $G$ is an absolute ideal of G. E. Fried formulated the problem of studying of abelian groups for which the converse is true, that is every absolute ideal is a fully invariant subgroup. Such groups are called “af i-groups”. The aim of this talk is to describe RAI-groups and af i-groups in the class of abelian Murley groups.

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Graphs associated with groups
Mark Lewis, Kent State University

There are many different graphs that have been associated with groups. In our talk, we consider the commuting graph and several related graphs. In particular, we will survey the known results regarding these graphs. We will present a new result that extends to Parker’s result on the commuting graphs of centerless solvable groups. We will present some new results on the cyclic graph (a.k.a. the enhanced power graph).

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A new criterion for solvability of a finite group

Patrizia Longobardi, Università di Salerno

The problem of detecting structural properties of a finite group by looking at some functions related to the structure of $G$ has been recently considered by various authors. For instance, H. Amiri, S.M. Jafari Amiri and I.M. Isaacs introduced the function

$$\psi(G) = \sum_{x \in G} o(x),$$

where $o(x)$ denotes the order of the element $x$. Many results concerning the function $\psi(G)$ have been obtained by many authors. Other functions have been introduced and studied, for instance

$$\rho(G) = \prod_{x \in G} o(x),$$

the product of element orders of a finite group $G$. Now we are concerned with another function related to the structure of a finite group $G$. We write

$$\sigma_1(G) = \frac{1}{|G|} \sum_{H \leq G} |H|.$$

Answering to a problem posed by M. Tǎrnăuceanu, together with Marcel Herzog and Mercede Maj, we have shown that, if $\sigma_1(G) < \frac{17}{20}$, then $G$ is solvable.

The aim of my talk is to provide a brief survey and outline this proof.

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Degrees of characters in the principal block
J. MIQUEL MARTÍNEZ, University of Valencia

Let $G$ be a finite group and let $p$ be a prime. The set of complex irreducible characters in the principal $p$-block of $G$ is rich enough that their degrees encode information of the structure of the group $G$. We study the case where the set of degrees of characters in the principal $p$-block of $G$ has size at most 2, finding information about the structure of $G$ and its Sylow $p$-subgroups. We will also show some related results on similar problems for arbitrary $p$-blocks.

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On the order of products of elements in finite groups
JUAN MARTÍNEZ MADRID, Universitat de València

It was proved by B. Baumslag and J. Wiegold that a finite group $G$ is nilpotent if and only if $o(x)o(y) = o(xy)$ for every pair of elements $x, y$ of coprime order. In this talk, we will present several theorems that generalize this result.

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On products of $\pi$-decomposable groups and Hall-like theorems
ANA MARTÍNEZ-PASTOR, Universitat Politècnica de València

Let $\pi$ a set of odd primes and let $G = AB$ a finite group which is the product of two $\pi$-decomposable subgroups $A = A_\pi \times A_\pi'$ and $B = B_\pi \times B_\pi'$. We discuss in this talk some Hall-like theorems for such a factorized group $G$ and, in particular, we show that $G$ has a unique conjugacy class of Hall $\pi$-subgroups, and any $\pi$-subgroup is contained in a Hall $\pi$-subgroup (i.e. $G$ satisfies property $D_\pi$).

(Joint work with Lev S. Kazarin and M. Dolores Pérez-Ramos.)

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Chains of normalizers of subnormal subgroups (part 2)
RYAN McCULLOCH, Elmira College

Given a subnormal subgroup $H$ of $G$, there is a standard subnormal series for $H$ in $G$ which is obtained by repeatedly taking normal closures. It can be shown that the length of the standard subnormal series for $H$ in $G$ is as small as possible. There is another way to define a subnormal series for a subnormal subgroup $H$ of $G$. One can form a subnormal series by taking the unique largest subnormal subgroup contained in successive normalizers. We call this the greedy series for a subnormal subgroup $H$ of $G$. We show that one can build a group $G$ with a subnormal subgroup $H$ of $G$ so that the length of the greedy series for $H$ in $G$ is as large as we like, while the length of the standard subnormal series for $H$ in $G$ is small. Our construction uses the wreath product.

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Products of classes which are a union of two classes
CARMEN MELCHOR, Universitat de València

There are several results about non-simplicity, solvability and normal structure of finite groups related to the product of conjugacy classes. In this framework it is well-known the Arad-Herzog’s conjecture which asserts that a finite group having two classes such that its product is again a conjugacy class is not a non-abelian simple group. Now, we fix our focus on the case when the product of two classes is the union of two conjugacy classes.

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Series of a construction related to the non-abelian tensor square
of groups

CARMINE MONETTA, University of Salerno

The non-abelian tensor square $G \otimes G$ of a group $G$, as introduced by Brown and Loday following Miller and Dennis, is defined to be the group generated by all symbols $g \otimes h$, with $g, h \in G$, subject to the relations

$$gg_1 \otimes h = (g^{g_1} \otimes h^{g_1})(g_1 \otimes h) \quad \text{and} \quad g \otimes hh_1 = (g \otimes h_1)(g^{h_1} \otimes h^{h_1})$$

for all $g, g_1, h, h_1 \in G$. Despite the easy presentation, it could be difficult to understand what group $G \otimes G$ is. Therefore one may deal with $G \otimes G$ studying related constructions, as done by Rocco, who considered the following operator $\nu$ in the class of groups. Let $G$ be a group and let $G^\varphi = \{g^\varphi \mid g \in G\}$ be an isomorphic copy of $G$ via $\varphi$. Then define the group

$$\nu(G) := \langle G \cup G^\varphi \mid [g_1, g_2^\varphi]^{g_3} = [g_1^{g_3}, (g_2^{g_3})^\varphi] = [g_1, g_2^{g_3\varphi}, g_i \in G]\rangle.$$

The motivation to study the group $\nu(G)$ is the commutator connection: indeed the subgroup $[G, G^\varphi]$ of $\nu(G)$ is isomorphic to $G \otimes G$. Hence one can investigate the structure of $\nu(G)'$ applying commutator calculus to grasp information about $G \otimes G$. In this talk we will present new results concerning the commutator structure of $\nu(G)$. In particular, we will show that the derived subgroup $\nu(G)'$ is a central product of three copies of the non-abelian tensor square $G \otimes G$, which will give in turn a new description for the derived and lower central series of the group $\nu(G)$. Our proofs involve both commutator calculus and techniques concerning universal properties of the non-abelian tensor square of groups.

Joint work with Raimundo Bastos, Ricardo De Oliveira, and Noraí Rocco.

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Hochschild transfer is group transfer
SHUCHEN MU, Binghamton University

It was proved that Hochschild homology of a group ring splits into direct sum of group homology of centralizers. When given a finite index subgroup, we can define transfer map on Hochschild homology which becomes group homology transfer on corresponding summands. We will present the proof on chain complex level and the problem becomes a group theoretical one.

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Computing the Frattini subgroup of a polycyclic group
MATTHIAS NEUMANN-BROSIG, TU Braunschweig, Germany and IAV GmbH

A first algorithm to compute the Frattini subgroup of a polycyclic-by-finite group was introduced by Baumslag, Cannonito, Segal & Robinson (1991). Here we present a new method to determine the Frattini subgroup of a polycyclic group. This new method has the advantage that it can be implemented and is practical for a variety of applications.

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The Muller-Schupp theorem for special monoids
CARL-FREDRIK NYBERG BRODDA, University of East Anglia

The study of ‘special’ monoids goes back to S. I. Adian and his student G. S. Makanin, who reduced the word problem of such monoids to the same problem for their group of units. There are many ways in which special monoids behave like “generalised” groups, and studying them has many similarities with combinatorial group theory. I will give a brief overview of the main ideas involved in studying special monoids – including the definition! – and the role played by the group of units. I will then present some recent full generalisations of the celebrated Muller-Schupp theorem from virtually free groups to special monoids with virtually free groups of units.

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Groups of small order type  
XUEYU (EILEEN) PAN, Monash University

For my MPhil project I investigate groups whose orders factorise into few primes. Theoretical classifications of these groups exist in the literature, but it is often difficult to extract the results. The aim of my thesis is to describe these classifications in a unified, modern language. An important and practical aspect of our new descriptions is that they lead to new efficient construction and determination algorithms for groups of some small order types.

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On the computability of the abelian kernel of an inverse semigroup  
VICENT PÉREZ CALABUIG, Universitat de València

The problem of computing the abelian kernel of a finite semigroup was first solved by Delgado describing an algorithm which decides whether a given element of a finite semigroup $S$ belongs to the abelian kernel. Steinberg extended the result for any variety of abelian groups with decidable membership. In this presentation, we use a completely different approach to complete these results by giving an exact description of the abelian kernel of an inverse semigroup. An abelian group which provides this abelian kernel is also constructed. As a consequence, the proabelian closure of a finitely generated subgroup of a free group is provided by means of our computation of the abelian kernel. We are working on extending this study to obtain results on the computability of generalised kernels of inverse semigroups, such as the metabelian kernel or the $\mathfrak{F}$-kernel, for extension-closed varieties of groups $\mathfrak{F}$.

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Congruences involving binomial coefficients arising from $p$-groups

JEFFREY RIEDL, University of Akron

Let $p$ be an arbitrary prime and let $n > 1$ be an integer. Let $W$ be the regular wreath product $C \wr H$ where $C$ and $H$ are cyclic groups of orders $p^2$ and $p^n$ respectively. Thus $W$ is the semidirect product $B \rtimes H$ where $B$ is the direct product of $p^n$ copies of $C$, with $H$ permuting these direct factors. Binomial coefficients arise naturally in the structure of the lower central series of $W$. Our study of $W$ has revealed some interesting congruences modulo $p^2$ involving binomial coefficients. We discuss these congruences.

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Blocks with few irreducible characters

NOELIA RIZO, University of the Basque Country

Let $G$ be a finite group, let $p$ be a prime number and let $B$ be a $p$-block of $G$ with defect group $D$. Studying the structure of $D$ by means of the knowledge of some aspects of $B$ is a main area in character theory of finite groups. Let $k(B)$ be the number of irreducible characters in the $p$-block $B$. It is well-known that $k(B) = 1$ if, and only if, $D$ is trivial. It is also true that $k(B) = 2$ if, and only if, $|D| = 2$. For blocks $B$ with $k(B) = 3$ it is conjectured that $|D| = 3$. When $B$ is the principal block, more can be said. For instance, the structure of defect groups of principal blocks having 3 and 4 irreducible characters is known. In this work we go one step further and analyze the structure of $D$ when $k(B) = 5$. This is a joint work with Mandi Schaeffer Fry and Carolina Vallejo.

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Character degrees in separable groups
LUCIA SANUS, Universitat de València

Under some separability conditions, many results on character degrees of finite groups can be unified in a single statement. We present a characterization, under some separability conditions, of when \( \text{Irr}_\pi(G) = \text{Irr}_\rho(G) \) where \( \pi \) and \( \rho \) are sets of primes and \( \text{Irr}_\pi(G) \) is the set of irreducible characters \( \chi \) of \( G \) such that all the primes dividing \( \chi(1) \) lie in \( \pi \). This generalizes the well-known theorems of Ito-Michler and Thompson on character degrees. This is joint work with G. Navarro and N. Rizo.

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Closed subgroups of the R. Thompson group \( F \)
MARK SAPIR, Vanderbilt University

Joint with Gili Golan. We prove that the closure of any finitely generated subgroup of \( F \) is finitely generated and undistorted in \( F \).

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Maximal subgroups of Thompson’s groups \( V \)
RACHEL SKIPPER, The Ohio State University

We will use graphs to build a family of maximal subgroups of Thompson’s group \( V \). This is a joint work with Jim Belk, Collin Bleak, and Martyn Quick.

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Determining synchronization of certain classes of primitive
groups of affine type

DUSTIN STORY, Colorado State University

The class of permutation groups includes 2-homogeneous groups, synchronizing groups, and primitive groups. Moreover, 2-homogeneous implies synchronizing, and synchronizing in turn implies primitivity. A complete classification of synchronizing groups remains an open problem. The search exists amongst the primitive groups, looking for examples of synchronizing and nonsynchronizing. Our main results are constructive proofs that show two classes of affine groups are nonsynchronizing.

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Laconic semigroups, laconic varieties, and the membership problem

ZORAN SUNIC, Hofstra University

Let $G = \langle A \rangle$ be a finitely generated group and $S$ a finitely generated subsemigroup of $G$, given by a finite list of group words over $A$ that generate $S$. The Membership Problem for $S$ in $G$ asks for an algorithm that, given a group word $w$ over $A$, decides if the element of $G$ represented by $w$ belongs to $S$. We recall a technique, upper distortion, introduced by Margolis, Meakin, and the speaker, that solves the Membership Problem in certain situations involving groups and subsemigroups, and we extend it beyond this limited context. (A semigroup $S$ is laconic if, for every finitely generated free semigroup $X^+$, every homomorphism $f : X^+ \to S$, and every element $s$ of $S$, the fiber $\phi^{-1}(s)$ is finite. In other words, every element of $S$ is represented by only finitely many words over $X$.)

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Restrictions on parameters of partial difference sets in nonabelian groups
ERIC SWARTZ, William & Mary

A partial difference set $S$ in a finite group $G$ satisfying $1 \not\in S$ and $S = S^{-1}$ corresponds to an undirected strongly regular Cayley graph $\text{Cay}(G, S)$. While the case when $G$ is abelian has been thoroughly studied, there are comparatively few results when $G$ is nonabelian. We provide restrictions on the parameters of a partial difference set that apply to both abelian and nonabelian groups and are especially effective in groups with a nontrivial center. In particular, these results apply to $p$-groups, and we are able to rule out the existence of partial difference sets in many instances. This is joint work with Gabrielle Tauscheck.

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Extending modules and related concepts
ADNAN TERCAN, Hacettepe University, Department of Mathematics

Let $R$ be a ring with identity and $M$ an right $R$-module. Recall that $M$ is called a right $CS$ (or extending or $C_1$) if every submodule of $M$ is essential in a direct summand of $M$. There are several generalizations of $CS$ notion. In this talk first we will give some basic properties of $CS$ modules then we deal with some of the generalizations of them.

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On a theorem of Schur
VIJI THOMAS, Indian Institute of Science Education and Research Thiruvananthapuram

We will discuss how the exponent of a central quotient of a group affects the exponent of the commutator subgroup. We will survey some results in this direction and then present our recent results. This is joint work with Komma Patali.

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Weak versions of extending modules  
Ramazan Yasar, Hacettepe University, Hacettepe-ASO 1.OSB  
Vocational School

Let $R$ be a ring with identity and $M$ an right $R$-module. Recall that $M$ is called a right Weak CS (or WCS) if every semisimple submodule of $M$ is essential in a direct summand of $M$. There are several generalizations of WCS notion. In this talk first we will give some well-known properties of WCS modules. Then we focus on some of the useful generalizations of them.

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Finiteness properties of normal subgroups of the Stein group $F_{2,3}$  
Matt Zaremsky, University of Albany (SUNY)

In this talk I will discuss recent joint work with Rob Spahn, in which we give a complete computation of all the Bieri-Neumann-Strebel-Renz invariants of the Stein group $F_{2,3}$. This is a relative of Thompson’s group $F$ that has proven over the years to be even more unusual than the famously unusual $F$. A consequence of our computation of the BNSR-invariants is a complete classification of the finiteness properties of every normal subgroup of $F_{2,3}$. In particular we prove that the kernel of every map from $F_{2,3}$ to $\mathbb{Z}$ is finitely presented, and even of type $F_\infty$, but there exist maps from $F_{2,3}$ to $\mathbb{Z}^2$ that are not even finitely generated. This makes the Stein group the first group with fully computed BNSR-invariants known to have this property. The talk will be self-contained, and no prior knowledge of any of the above topics will be needed.

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