HOW BIG IS EPSILON?

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A 1979 theorem of Chapman-Ferry says that if M is a compact connected topological n-manifold[1] without boundary with topological metric d, then there is an $\epsilon > 0$ so that if $f : M \to N$ is a map from M to a connected manifold of the same dimension such that $\text{diam}(f^{-1}(x)) < \epsilon$ for every $x \in N$, then f is homotopic to a homeomorphism.

This theorem and its descendants play a continuing role in the work of Farrell-Jones, Bartels-Lück, and others on the Novikov, Borel, and Farrell-Jones Conjectures, the general strategy being to apply ideas from dynamics to "squeeze" a given homotopy equivalence to an appropriately "controlled" equivalence to which some version of the theorem quoted above applies.

We will show that the behavior of ϵ in our old theorem depends on results from algebraic topology on the vanishing of the K-homology of Eilenberg-MacLane spaces of torsion groups. An application to computational topology is suggested.

This is joint work with Alexander Dranishnikov and Shmuel Weinberger.

[1]Chapman-Ferry did the cases $n \geq 5$. The case $n = 4$ is due to Freedman-Quinn and n=3 follows from work of Perelman