Problem 7. n people are standing around a circle. Each person made an appointment with the banker to make a transaction, either a withdrawal or a deposit. The total amount of all the withdrawals is going to be the same as the total amount of all the deposits. The banker arrives without any money. Show that the banker can perform all the transactions by choosing the first person and then going around the circle.

Solution. Number the people $1, 2, ..., n$ going clockwise around the circle. Let a_i be the amount of the transaction of the i-th person (a_i is negative when the transaction is a withdrawal). For $m > n$ define a_m to be the same as a_{m-n}. Now consider the sums $s_m = a_1 + \ldots + a_m$, $m = 1, 2, \ldots$ and set $s_0 = 0$. Since $s_n = 0$, the sequence (s_m) is periodic with period n. Thus there is k such that $0 \leq k \leq n-1$ and s_k is the smallest value of the sequence (s_m). The banker should start with person number $k+1$ and go around the circle clockwise. Then, for any i, after seeing the first i customers the banker will have $a_{k+1} + \ldots + a_{k+i} = s_{k+i} - s_k \geq 0$ at his disposal. Thus the banker can perform all the transactions (will never run out of money).

Exercise. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a bounded periodic function with period l which is integrable on the interval $[0, l]$. Let $M = (\int_0^l f(t)\mathrm{d}t)/l$. Prove that there is u such that $\int_u^x f(t)dt \geq M(x-u)$ for every x.

Question. How is the above exercise related to Problem 7?