§3.9 Antiderivatives

In each sentence, find a function that fills in the blanks, if you can. (For some of them, you won't be able to.)

a) _____ is an antiderivative of $\cos(x)$ because $\frac{d}{dx}(\_\_\_\_) = \cos(x)$.

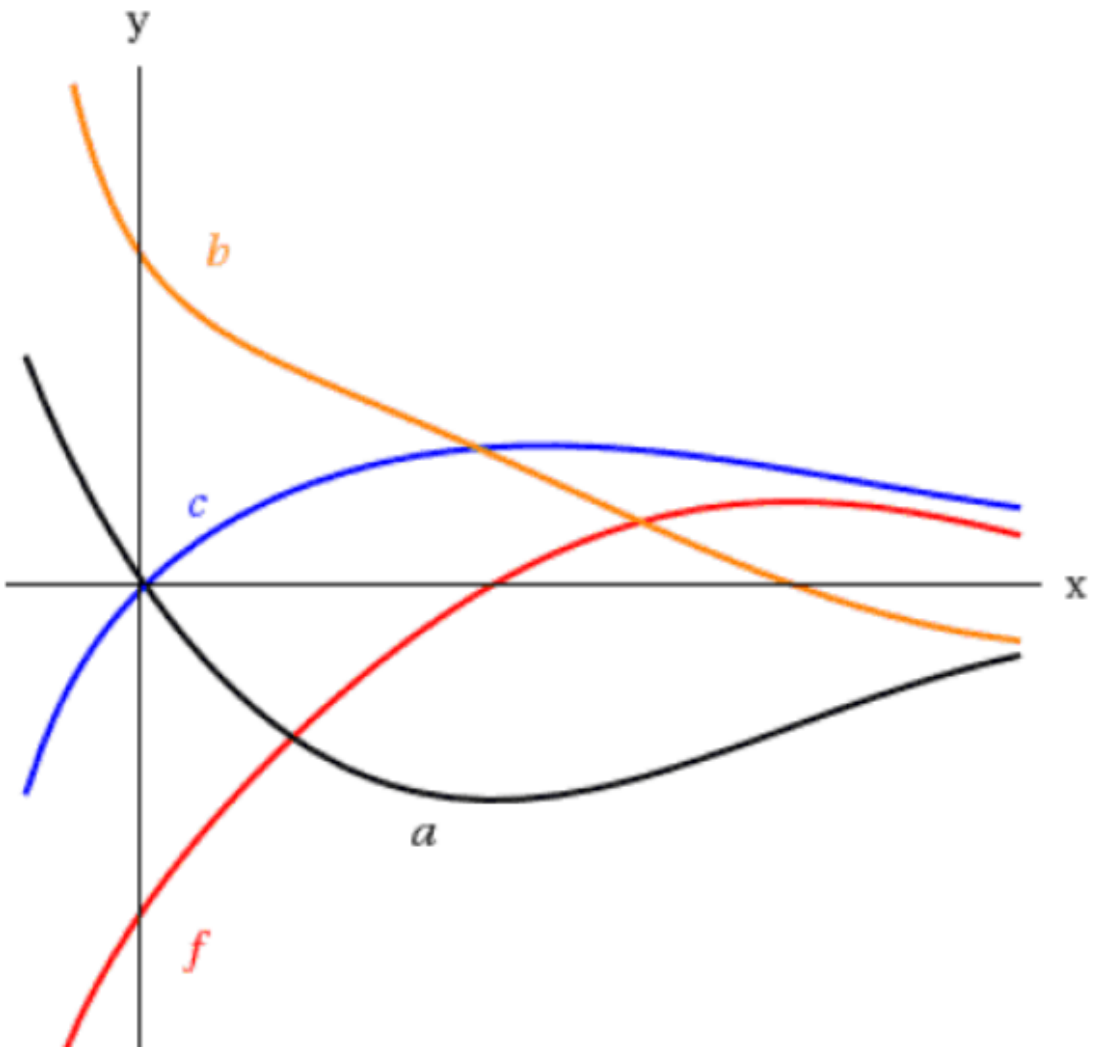
b) _____ is an antiderivative of $\tan(x)$ because $\frac{d}{dx}(\_\_\_\_) = \tan(x)$.

c) _____ is an antiderivative of $x\cos(x)$ because $\frac{d}{dx}(\_\_\_\_) = x\cos(x)$.

d) _____ is an antiderivative of $x^3 + 2$ because $\frac{d}{dx}(\_\_\_\_) = x^3 + 2$.

e) _____ is an antiderivative of $\sqrt{6x+1}$ because $\frac{d}{dx}(\_\_\_\_) = \sqrt{6x+1}$.

Which function from $\{a, b, c\}$ is an antiderivative of f?



True or **False**

An antiderivative of a sum of functions, $f + g$, is an antiderivative of f plus an antiderivative of g.

An antiderivative of a product of functions, fg, is an antiderivative of f times an antiderivative of g.

Find the most general antiderivative of each function.

a) $f(x) = \frac{1}{2}x^2 - 2x + 6$

b) $g(x) = (x + 5)(2x - 6)$

c) $h(x) = \frac{3 + t + t^2}{\sqrt{t}}$

Let f be a function that satisfies $f''(x) = 12x + \sin(x)$.

a) If you know nothing else about f, give the best formula you can for f.

b) If you know $f'(\pi) = 1$, give the best formula you can for f.

c) If you know $f'(\pi) = 1$, and $f(\pi) = 0$, give the best formula you can for f.

Suppose you are given the acceleration function $a(t)$ of an object. Let's say you are given that $v(0) = 1$.

True or **False**

You can find the position of the object at any time t.

In each of the following, a particle is moving with the given data. Find the position function of the particle.

a) $v(t) = 1.5\sqrt{t}$, $s(16) = 67$.

b) $a(t) = 2t + 5$, $s(0) = 2$, $v(0) = -5$.

Find f if $f''(\theta) = \sin(\theta) + \cos(\theta)$, $f(0) = 3$, and $f'(0) = 3$.

Find f if $f'''(x) = \cos(x)$, $f(0) = 5$, $f'(0) = 1$, and $f''(0) = 8$.

Let $f(x) = \dfrac{1}{x^2}$.

True or **False**

If $F(x)$ is an antiderivative of f with the property $F(1) = 1$, then $F(-1) = 3$.

Find a function f such that $f'(x) = 2x^3$ and the line $2x + y = 0$ is tangent to the graph of f.

A stone was dropped off a cliff and hit the ground with a speed of 112 ft/s. What is the height of the cliff? (Use 32 ft/s^2 for the acceleration due to gravity.)

What constant acceleration is required to increase the speed of a car from 25 mi/h to 53 mi/h in 3 s?

If a diver of mass m stands at the end of a diving board with length L and linear density ρ, then the board takes on the shape of a curve $y = f(x)$, where

$$Eky'' = mg(L - x) + \frac{1}{2}\rho g(L - x)^2.$$

E and k are positive constants that depend on the material of the board and g (< 0) is the acceleration due to gravity.

a) Find an expression for the shape of the curve.

b) Use $f(L)$ to estimate the distance below the horizontal at the end of the board.