§3.7 Optimization

Find two numbers whose difference is 140 and whose product is a minimum.

Find the dimensions of a rectangle with perimeter 60 meters whose area is as large as possible.

Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 feet wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

a) Draw several diagrams to illustrate the situation, some short boxes with large bases and some tall boxes with small bases. Find the volumes of several such boxes.

b) Draw a diagram illustrating the general situation. Let x denote the length of the side of the square being cut out. Let y denote the length of the base.

c) Write an expression for the volume V in terms of x and y.

d) Use the given information to write an equation that relates the variables x and y.

e) Use part (d) to write the volume as a function of x.

f) Finish solving the problem by finding the largest volume that such a box can have.

A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of this base is twice the width. Material for the base costs \$5 per square meter. Material for the sides costs \$3 per square meter. Find the cost of materials for the cheapest such container.

(Round your answer to the nearest cent.)

A manufacturer has been selling 1000 flat-screen TVs a week at \$350 each. A market survey indicates that for each \$10 rebate offered to the buyer, the number of TVs sold will increase by 100 per week.

a) Find the demand function (price p as a function of units sold x).

b) How large a rebate should the company offer the buyer in order to maximize its revenue?

c) If its weekly cost function is

$$C(x) = 60,000 + 120x$$

how should the manufacturer set the size of the rebate in order to maximize its profit?

A boat leaves a dock at 1 PM and travels due south at a speed of 20 km/hour. Another boat has been heading due east at 15 km/hour and reaches the same dock at 2 PM. How many minutes after 1 PM were the two boats closest together?

At which points on the curve

$$y = 1 + 40x^3 - 3x^5$$

does the tangent line have the largest slope?

A piece of wire 30m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle.

a) How much wire should be used for the square in order to maximize the total area?

b) How much wire should be used for the square in order to minimize the total area?

A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter x of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 32 feet, find the value of x so that the greatest possible amount of light is admitted.

A designer wants to introduce a new line of bookcases. He wants to make at least 100 bookcases, but not more than 2000 of them. He predicts the cost of producing x bookcases is $C(x)$. Assume that $C(x)$ is a differentiable function. Which of the following must he do to find the minimum average cost,

$$k(x) = \frac{C(x)}{x}?$$

i) Find the points where $k'(x) = 0$ and evaluate $k(x)$ there
ii) Compute $k''(x)$ to check which of the critical points in (i) are local maxima.
iii) Check the values of k at the endpoints of its domain.

a) i only
b) i and ii only
c) i and iii only
d) i, ii, and iii

The rate (in appropriate units) at which photosynthesis takes place for a species of phytoplankton is modeled by the function

$$P(n) = \frac{120n}{n^2 + n + 4}$$

where n is the light intensity (measured in thousands of foot-candles). For what light intensity is P a maximum?

What is the maximum vertical distance between the line

$$y = x + 6$$

and the parabola

$$y = x^2$$

for $-2 \leq x \leq 3$?

Find the area of the largest rectangle that can be inscribed in the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

An oil refinery is located on the north bank of a straight river that is 1 kilometer wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 6 kilometers east of the refinery.
The cost of laying pipe is $\$400,000$ per kilometer over land to a point P on the north bank and $\$800,000$ per kilometer under the river to the tanks. To minimize the cost of the pipeline, how far downriver from the refinery should the point P be located?

Suppose now that the refinery is located 1 kilometer north of the north bank of the river. Now where should P be located?