§3.1 Critical Points and the Closed Interval Method

Sketch the graph of $y = (x-1)^2 + 2$ on the closed interval $[-4, 4]$.

a) What are the local maximum and minimum values? points?

b) What are the absolute maximum and minimum values? points?

Find the critical numbers of the following functions

a) $f(x) = 8x^3 - 12x^2 - 48x$

b) $g(x) = x^{\frac{3}{4}} - 9x^{\frac{1}{4}}$

c) $h(\theta) = 18\cos(\theta) + 9\sin^2(\theta)$

Show that 5 is a critical number of the function

$$g(x) = 2 + (x - 5)^2$$

but g does not have a local extreme value of 5.

If f has a minimum value of c, does the function $g(x) = -f(x)$ have a maximum value of c?

Let $f(x)$ be a differentiable function on a closed interval with $x = a$ being one of the endpoints of the interval. If $f'(x) > 0$ for all x, then

a) f could have either an absolute maximum or minimum at $x = a$.

b) f cannot have an absolute maximum at $x = a$.

c) f must have an absolute minimum at $x = a$.

d) $x = a$ must be a critical number for f.

If f is continuous on $[a, b]$, then

a) There must be local extreme values, but there may or may not be an absolute maximum or minimum value for the function.

b) There must be numbers m and M such that $m \leq f(x) \leq M$, for all x in $[a, b]$.

c) Any absolute maximum or minimum would be at either the endpoints of the interval, or at places in the domain where $f'(x) = 0$.

Find the absolute extrema of:

a) $f(x) = x^3 - 3x + 1$ on the interval $[0, 3]$.

b) $g(x) = \dfrac{x^2 - 4}{x^2 + 4}$ on the interval $[-4, 4]$.

c) $h(t) = t\sqrt{4 - t^2}$ on the interval $[-1, 2]$.

d) $k(x) = x + \cot\left(\frac{x}{2}\right)$ on the interval $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$.

Find the highest and lowest points on the graph of $f(x) = x^3 - 3x + 6$ on the following intervals:

a) $[-2, 2]$

b) $[-2, 3]$

c) $(-2, 3)$

Show that the maximum and minimum values of the function

$$f(x) = x^3 + ax^2 + bx + c$$

on the interval $[p, q]$ occur at the endpoints if $a^2 < 3b$.

If a and b are positive numbers, find the maximum value of

$$f(x) = x^a(1 - x)^b$$

on the interval $[0, 1]$.