§2.6 Functions Defined Implicitly and their Derivatives

Draw a graph of $x = \sin(y)$ and find the slope of the line tangent to the graph at the point $(0, \pi)$.

Find dx/dy and dy/dx if $y \sec(x) = 6x \tan(y)$.
Explain the difference between these expressions.

Find dy/dx using an implicitly defined function ("implicit differentiation").

a) $x^4 + y^3 = 1$

b) $7x^2 + 5xy - y^2 = 6$

c) $x^7(x + y) = y^2(4x/y)$

d) $4\cos(x)\sin(y) = 2$

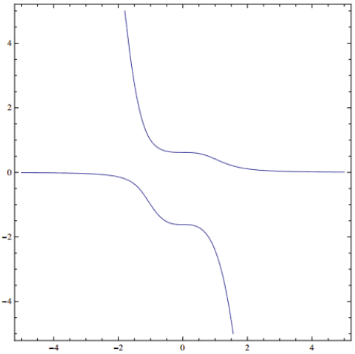
e) $5y\sin(x^2) = 9x\sin(y^2)$

f) $\sqrt{7x + y} = 6 + x^2y^2$

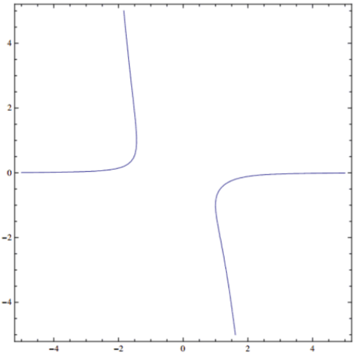
Explain (without calculating) why the two following equations will yield the same formula for dy/dx. Does this mean that the two graphs will have exactly the same tangent lines?

$$x^3y + y^2 + y = 1$$
$$x^3y + y^2 + y = -1$$

ContourPlot[x^3 y + y^2 + y == 1, {x, -5, 5}, {y, -5, 5}]



ContourPlot[x^3 y + y^2 + y == -1, {x, -5, 5}, {y, -5, 5}]



Find an equation of the tangent line to the **ellipse**

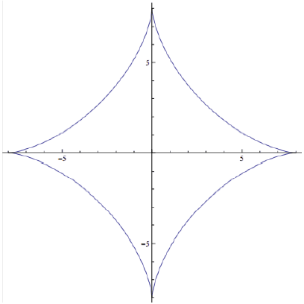
$$9x^2 + xy + 9y^2 = 19$$

at the point $(1, 1)$.

Find an equation of the tangent line to the **astroid**

$$x^{2/3} + y^{2/3} = 4$$

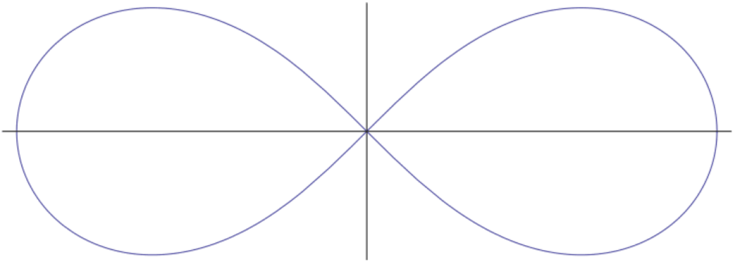
at $(-3\sqrt{3}, 1)$.



Find the points on the **lemniscate**

$$8(x^2 + y^2)^2 = 25(x^2 - y^2)$$

where the tangent is horizontal.



If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, find $f'(1)$.

Find dx/dy and dy/dx and dz/dx if

$$y \sec(z) = 6x \tan(y).$$

Find y'' by using an implicitly defined function ("implicit differentiation").

$$4x^2 + y^2 = 9$$

When we introduced the Power Rule, we explained it for $y = x^n$ when n is a nonnegative integer, and we promised that later we'd explain it when n is a rational and/or negative number. The moment has come. In the following, you should use the Power Rule **only for n a nonnegative integer** to prove it the Power Rule for all rational numbers.

a) Warm-up: write $y = x^{\frac{2}{3}}$ as $y^3 = x^2$. Then use Implicit Differentiation to show $y' = \frac{2}{3}x^{-\frac{1}{3}}$.

b) Let $y = x^{\frac{p}{q}}$, where p and q are positive integers. Use the same method as the previous problem to show $y' = \frac{p}{q}x^{\frac{p}{q}-1}$.

c) Warm-up: write $y = x^{-1}$ as $xy = 1$. Then use Implicit Differentiation to show $y' = -x^{-2}$.

d) Let $y = x^{-a}$, where a is a positive rational number. Use the same method as the previous problem to show $y' = -ax^{-a-1}$.

When you solve for y' (or $f'(x)$) in an implicit differentiation problem, you have to solve a quadratic equation

a) Always

b) Sometimes

c) Never

Find equations of both the tangent lines to the ellipse

$$x^2 + 9y^2 = 81$$

that pass through the point $(27, 3)$.

The Thin Lens Equation in optics relates the focal length f of a lens, the distance a from an object to the lens, and the distance b from the object's image to the lens. The equation is

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}.$$

Let's say you have a lens with focal length 10 cm.

a) Which of the following derivatives describes the rate at which the position of the image changes as you move the object?

$$\frac{da}{db} \qquad \frac{db}{da} \qquad \frac{da}{df} \qquad \frac{df}{da}$$

b) If the object is 20 centimeters from the lens and moving away from the lens, where is the object's image and in what direction is it moving?