§2.2 The Derivative Function

Is the function

$$f(x) = \begin{cases} 2 - x & \text{if } x \leq 2 \\ x^2 - 4x + 4 & \text{if } x > 2 \end{cases}$$

differentiable at 2?

Find all a and b such that the function

$$g(x) = \begin{cases} 2 - x & \text{if } x \leq 2 \\ x^2 + ax + b & \text{if } x > 2 \end{cases}$$

is differentiable for all x.

You are designing the first ascent and drop for a roller coaster. You want the slope of the ascent to be .8 and the slope of the drop to be -1.6. You will connect these two straight stretches by part of a parabola

$$y = ax^2 + bx + c$$

of width 100 units.

a) Certainly you don't want a sharp corner in your tracks at the points where the linear parts meet the parabola. This puts a condition on the tangent lines of the parabola – what's the condition?

b) Find a formula for the parabola.

If $f + g$ is differentiable at a, are f and g necessarily differentiable at a?

If $f'(a)$ exists, then $\lim_{x \to a} f(x)$

a) must exist, but there is not enough information to determine it exactly

b) equals $f(a)$

c) equals $f'(a)$

d) may not exist

A slow freight train chugs along a straight track. The distance it has traveled after x hours is given by a function $f(x)$. An engineer is walking along the top of the box cars at the rate of 3 miles per hour in the same direction as the train is moving. The speed of the man relative to the ground is

a) $f(x) + 3$

b) $f'(x) + 3$

c) $f(x) - 3$

d) $f'(x) - 3$

Use the definition of the derivative and the properties of limits to compute the derivatives of the following functions:

a) $f(x) = \dfrac{1-x}{2+x}$

b) $g(x) = \dfrac{1}{\sqrt{x-2}}$