

§1.8 Continuity

Where are the following functions continuous?

a) $f(x) = \frac{\sqrt{x}}{1 + \sin(x)}$

d) $b(x) = \frac{1}{|x - 2|}$

b) $g(x) = (\sec(x))^2 + x$

e) $c(x) = \frac{1}{|x - 2| + 1}$

c) $a(x) = \frac{x}{|x|}$

f) $e(x) = \frac{1}{1 + \sqrt{x}}$

Let $P(t)$ be the cost of parking in New York City's parking garages for t hours. So,

$$P(t) = \$20 \text{ per hour or fraction thereof}$$

For example, if you are in the garage for two hours and one minute, you pay \$60. Graph the function P and discuss the continuity.

True or False

If t_0 closely approximates some time, T , then $P(t_0)$ closely approximates $P(T)$. Be prepared to justify your answer.

You decide to estimate π^2 by squaring longer decimal approximations of $\pi = 3.14159\dots$. Choose which of the following can be justified with what you've learned so far:

- a) This is a good idea because π is a rational number.
- b) This is a good idea because $f(x) = x^2$ is a continuous function.
- c) This is a bad idea because π is irrational.
- d) This is a good idea because $f(x) = \pi^x$ is a continuous function.

Define the function

$$f(x) = \begin{cases} 1 + x^2 & \text{if } x \leq 0 \\ 4 - x & \text{if } 0 < x \leq 4 \\ (x - 4)^2 & \text{if } x > 4. \end{cases}$$

- ▶ Where is f continuous?
- ▶ At the points where it's not continuous, state whether it's continuous from the left, from the right, or neither.
- ▶ AFTER you've done this, sketch the graph of f .

Find all values a such that the function

$$g(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x + a & \text{if } x > 1. \end{cases}$$

is continuous.

Use the Intermediate Value Theorem to show that the equation

$$x^4 + x - 4 = 0$$

has a root in the interval $(1, 2)$.

Argue using the Intermediate Value Theorem that my hair was 6 inches long at some point in the past. If I boast that my beard was once over a foot long, would I be able to use the Intermediate Value Theorem and my present beard length as proof of my claim?