Final Exam (Version A)

Math in action

Spring 2014

Wednesday, May 14th

Name (printed): Solutions

Signature: _____

Section number: _____

Directions:
The test is two hours long. No phone, calculator, electronics, notes, talking to friends, etc. You may use only a pen or pencil. Absolutely no cheating!

No scrap paper! If you need some you may use the back side of this exam or ask someone who is proctoring the exam.

Read carefully. Show your work. Check your work. The test is out of 200 points.

Do not turn the page until the professor and/or TA's say so.

Do not write below this line.

	Points		Points		Points
1		7		13	
2		8		14	
3		9			
4		10			
5		11			
6		12			

Total =

Problem 1(10 points) Suppose some people named Arya, Balon, Cersei, Daenerys, Eddard, and Farlen want to divide a pizza using claim and challenge. They play in order from A to F. In the first round, person E gets a slice. In the second round, everyone remaining challenges. In the third round, only B challenges. No person challenges in any other round.

A B C D E F
4 3 5 6 1 2

(2 points)
(a) What is the maximum number of people that could have challenged in the first round?

4

(2 points)
(b) Who gets a slice in the second round?

F

(3 points)
(c) Who gets a slice in the fourth round?

A

(3 points)
(d) Who gets the final slice?

D

Problem 2 (15 points)
30 new desks are to be divided up between five schools, A, B, C, D, and E. Use Hamilton's method to distribute the desks amongst the schools.

School	A	B	C	D	E	Total
Enrollment	40	56	64	79	61	300
No. of desks: 30			Standard divisor: 10			
Exact Quota	4	5.6	6.4	7.9	6.1	XXXXX
Lower Quota	4	5	6	7	6	28
Frac Part	0	.6	.4	.9	.1	XXXXX
Surplus		1		1		2
Total	4	6	6	8	6	30

96
64
160
140

Problem 3 (15 points)
Consider a deck of cards that has 11 ranks (1 through 11) and five suits (hearts, diamonds, clubs, spades, and moons). Thus there are 55 cards in the deck. (You may leave your answers in terms of products and sums of factorials, ${}_nP_k$, ${}_nC_k$ etc. For example, answers of the form 6^5 or ${}_6C_3 \times {}_3C_1$ are acceptable.)

(3 points) **(a)** Suppose we draw a **5-card** hand from this deck. How many flushes are possible? (A flush is a hand where all 5 cards are of the same suit, and for this question, a straight-flush counts as a flush.)

${}_5C_1 \times {}_{11}C_5$
suit ranks

(3 points) **(b)** Suppose we draw a **5-card** hand from this deck. How many straight-flushes are possible?(A straight-flush is a flush where the ranks of the cards are in order, for example, 3,4,5,6,7 of hearts.)

${}_5C_1 \times {}_7C_1$
suit ranks

(3 points) **(c)** Suppose we draw a **5-card** hand from this deck. What is the **probability** of a full-house? (A full-house has three cards of one rank, and two cards of another rank, for example, 4,4,4,7,7).

$$\frac{{}_{11}C_1 \times {}_5C_3 \times {}_{10}C_1 \times {}_5C_2}{{}_{55}C_5}$$

(3 points) **(d)** Suppose we draw a **5-card** hand from this deck. A hand is called *small* if the sum of the ranks of the cards in the hand is 5. How many *small* hands are there?

1

(3 points) **(e)** Suppose we draw a **6-card** hand from this deck. How many *small* 6-card hands are there? (Note: *small* is still defined as in part (d), meaning the sum of the ranks is 5).

0

Problem 4 (15 points)

(5 points) Six

(a) Five people (labeled A through F, in order) are in this weighted voting system: [16; 8, 5, 3, 2, 1, 1]. Which of the voters in the coalition {A, B, C} are critical?

all: A, B, C

(3 points)

(b) List all possible values for q that satisfy the quota restriction for the weighted voting system [q ; 7, 3, 1, 1].

$6 < q \leq 12$

(5 points)

(c) Consider the weighted voting system: [q ; 7, 3, 1, 1]. Is there a possible q value satisfying the quota restriction for which this system has a dictator? If so, give an example of such a q-value. If not, say "no."

yes: $q = 7$

(2 points)

(d) Give an example of a q value so that [q ; 7, 3, 1, 1] is anarchy.

$q = 1$

Problem 5(10 points)

(3 points)

(a) Using the following preference schedule, how many points would C receive if we were to use the Borda count method to decide on a winner in this election? You do **not** need to carry out the whole election.

	Num ballots			
Choice	9	6	9	2
1st	C	C	D	B
2nd	D	A	B	A
3rd	A	D	C	D
4th	B	B	A	C

$15 \times 4 + 9 \times 2 + 2 \times 1$

(2 points)

(b) Using the preference schedule given in part (a), find how many points C would receive if the pairwise comparisons method is used. You do **not** need to carry out the whole election.

Actually, we don't even need to do pairwise: C has a majority & hence beats everyone head-to-head & gets 5 points

(2 points)

(c) Using the plurality method, which candidate would win the election?

C

(3 points)

(d) Suppose you have some other voting method that claims candidate B is the winner. Would any fairness criteria be violated by this result? If so, explain.

yes - majority crit.

Problem 6 (15 points)
Marge, Carol, Jeff, and Blake have inherited some valuable pies from a distant relative. Carry out the division of these objects between them using the sealed bids method.

	Marge	Carol	Jeff	Blake
Apple Pie	\$600	\$1200	\$1100	\$500
Cherry Pie	\$600	\$400	\$900	\$1100
Lemon Pie	\$500	\$600	\$700	\$300
Lime Pie	\$1100	\$600	\$900	\$500
Total Value	2800	2800	3600	2400
Fair Share	700	700	900	600
Allocated	1100	1200	700	1100
Difference	-400	-500	200	-500
Surplus= 1200				
Surplus Share	300	300	300	300

Summary

Item(s)	lime	apple	lemon	cherry
Item's Value	1100	1200	700	1100
Cash	-100	-200	500	-200
Net Total	1000	1000	1200	900

Problem 7 (20 points)
Two 6-sided dice are rolled. One of the dice is fair and colored red, and the other is colored blue and weighted with weights according to the following table:

face of die	1	2	3	4	5	6
probability	2/6	1/6	1/6	1/6	1/6	0

(5 points)
(a) What is the probability that both dice land on 1?

$\frac{1}{6} \times \frac{2}{6}$

(5 points)
(b) Consider the event S = Both dice land on the same number. What is the probability of S?

$\left(\frac{1}{6} \times \frac{2}{6}\right) + \left(\frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times 0\right)$

(5 points)
(c) Consider the event E = The sum of the numbers shown on the red and blue dice is 1, 2, or 3. What is the probability of E?

$\underbrace{\left(\frac{1}{6} \times \frac{2}{6}\right)}_{1+1} + \underbrace{\left(\frac{1}{6} \times \frac{1}{6}\right)}_{1+2} + \underbrace{\left(\frac{1}{6} \times \frac{2}{6}\right)}_{2+1}$

(5 points)
(d) Consider the event O = The sum of the numbers shown on the red and blue dice is 12. What is the probability of O?

$\frac{1}{6} \times 0 = 0$

Problem 8(15 points) Steve is very bored one day and decides to write down every possible 8-letter "word", where a "word" is **any** combination of the lower-case letters (a-z). Examples include "keyboard", and "rttttyzx".

(3 points)

(a) How many "words" will Steve write down?

26^8

(4 points)

(b) How many "words" will Steve write down that have no repeated letter?

$26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19$

(4 points)

(c) How many "words" will Steve write down that have at least one repeated letter?

$26^8 - (\text{part (b)})$

(4 points)

(d) How many "words" will Steve write down that start with the sequence "aaaaaa"?

26^2

Problem 9(10 points)A poll was taken among old ladies to determine the number of cats they owned.

(2 points) **(a)** Fill out the table.

Number of cats	0	1	2	3	4	5
frequency	17	33	10	15	10	15
cumulative freq.	17	50	60	75	85	100

(5 points) **(b)** Calculate the five number summary.

min=0 max=5 med = 1.5 Q1=1 Q3=3.5

location 50-51 loc. 25-26 loc. 75-76

(3 points) **(c)** Write down the mean. (You need not compute it.)

$$\frac{(0\times 17)+(1\times 33)+(2\times 10)+(3\times 15)+(4\times 10)+(5\times 15)}{100}$$

Problem 10 (10 points)

(8 points)

(a) Calculate the Standard Deviation of the data set 7,7,8,8,10. (You may leave your answer in terms of the Variance).

$$\mu = \frac{7+7+8+8+10}{5} = \frac{40}{5} = 8$$

$$\text{Var} = \frac{(7-8)^2+(7-8)^2+(8-8)^2+(8-8)^2+(10-8)^2}{5}$$

$$= \frac{6}{5}, \text{ so } \sigma = \sqrt{\frac{6}{5}}$$

(2 points)

(b) True or False: The standard deviation of the data set 1,1,1,1,1,1,1,1,1 is 9.

Problem 11(15 points) The scores on the Math 130 final are approximately normally distributed with mean $\mu = 75$ points, and standard deviation $\sigma = 8$ points.

(3 points)

(a) Approximately what percentage of the scores are higher than an 83?

16%

(3 points)

(b) What is the 84th percentile?

83

(3 points)

(c) About what percent of the scores are between 67 and 83?

68%

(3 points)

(d) Estimate the 25^{th} percentile score. (You do not need to compute the decimal).

$75 - .675 \times 8$

(3 points)

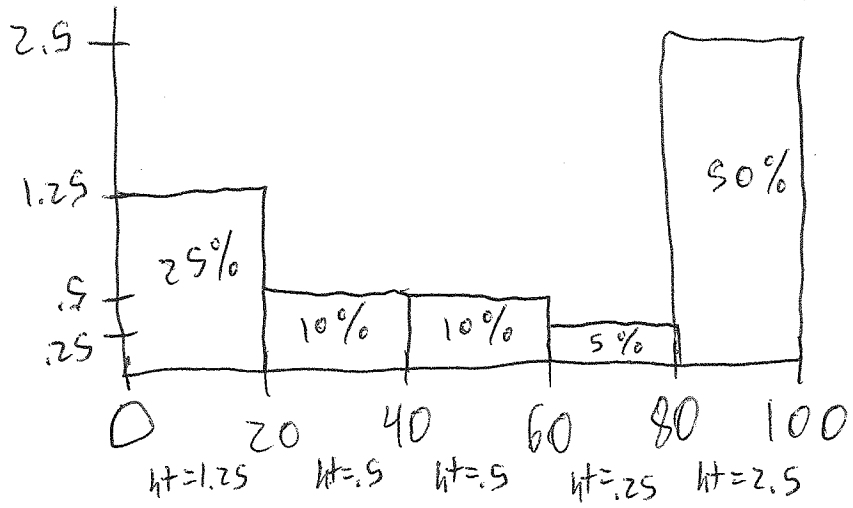
(e) About what percent of the scores are between 67 and 91?

$\frac{97-68}{2} = \frac{29}{2} = 14.5$ > so $68 + 14.5 =$ 82.5%

Problem 12 (15 points) Jeff Goldblum angrily stops 600 people on the streets and asks them for their age, and then records the results in this table. Create a histogram for the ages. (Note: $50/20 = 2.5$ and $25/20 = 1.25$)

total = 600

ages	[0, 20)	[20, 40)	[40, 60)	[60,80)	[80,100]
frequency	150	60	60	30	300
	25%	10%	10%	5%	50%



Problem 13(15 points) Some puppy scientists are trying to study the puppy population of Binghamton. They captured, tagged and released 400 puppies. A year later, they captured 300 puppies of which 30 were tagged.

(3 points)

(a) Find $\hat{p}$, which is the estimate for the fraction of the puppy population tagged.

$$\hat{p} = \frac{30}{300} = \frac{1}{10}$$

(3 points)

(b) Find the central estimate for the population.

$$\frac{1}{10} = \frac{400}{N} \Rightarrow N = 4000$$

(3 points)

(c) Give a formula for the standard error.

$$\text{st. error} = \sqrt{\frac{\hat{p} \times (1-\hat{p})}{n}} = \sqrt{\frac{\frac{1}{10} \times \frac{9}{10}}{300}}$$

not necessary to simplify fractions but makes part (d) cleaner

(6 points)

(d) Assume the standard error is .02. Find the 95% confidence interval for the population of puppies. You may leave the inequality/interval as fractions.

$$\frac{1}{10} - 2 \times .02 \leq \frac{400}{N} \leq \frac{1}{10} + 2 \times .02$$

$$\frac{400}{\frac{1}{10} - 2 \times .02} \geq N \geq \frac{400}{\frac{1}{10} + 2 \times .02}$$

40

Problem 14 (2 points each)

(a) **True** or **False** The median of a set of numbers is the average of its biggest and smallest number.

(b) **True** or **False** $100! > {}_{100}C_2$

(c) **True** or **False** Mario has a half chocolate, half vanilla cake worth \$50. If he likes chocolate nine time more than he likes vanilla, then he values the chocolate half at \$40.

(d) **True** or **False** The area of a histogram corresponds to the standard deviation of the data set.

(e) **True** or **False** The number 4.123 is rounded up to 5 using Adams' Method.

(f) **True** or **False** If a voter has a 100% power index, then that voter is a dictator.

(g) **True** or **False** The Independence of Irrelevant Alternatives Criterion states that If a non-winner candidate is removed from the ballot, and a recount is done, then the winner should be a different candidate.

(h) **True** or **False** The probability of flipping 55 heads in a row on a fair coin is less than the probability of rolling two fair six-sided dice such that their sum is 1. $\frac{1}{2^{55}} < 0$ no

(i) **True** or **False** If the person who recieves the most first place votes under a certain voting method loses the election, that voting method violates the Majority Criterion.

(j) **True** or **False** There are $n!$ possible preference ballots for an election involving n candidates.