Final Exam (form A)

Math in action

Spring 2013

Wednesday, May 15th

Name (printed): Solutions

Signature: ______________________

Section number: __________

Directions:

Two hours long. No phone, calculator, notes, neighbors, etc...only a pen or pencil is allowed. Absolutely no cheating!

Read carefully. Show your work. Check your work.

Do not turn the page until the professor and/or TA's say so.

Do not write below this line.

	Points		Points		Points
1		7		13	
2		8		14	
3		9			
4		10			
5		11			
6		12			

Total =

Problem 1
Use the preference schedule below:

	Num ballots			
Choice	8	4	6	2
1st	D	A	D	B
2nd	B	C	C	D
3rd	A	B	A	C
4th	C	D	B	A

(3 points) **(a)** If the Borda count method is used, how many points would B receive? (You need NOT carry out the entire election.)

$8 \times 3 + 4 \times 2 + 6 \times 1 + 2 \times 4$

(3 points) **(b)** If the pairwise comparisons method is used, how many points would A receive? (You need NOT carry out the entire election.)

AvB : 10-10 $A+\frac{1}{2}$, AvC : 12-8 A+1, AvD: 4-16 A+0 so 1.5

(2 points) **(c)** If plurality with elimination is used, who would win the election?

D (D has a majority, so we don't even need to eliminate anyone)

(2 points) **(d)** True or **False** If a certain voting method is used that produces a winner other than D, then the majority fairness criterion would be violated.

Problem 2 Consider the weighted voting system [–; 7, 3, 2, 1].
(2 points each)
(a) What is the total weight of this voting system?

13

(b) List a q-value that would be non-functional.

14

(c) List all possible q-values satisfying the quota restriction.

$\frac{13}{2} < q \leq 13$, so $\boxed{7, 8, 9, 10, 11, 12, 13}$

(d) Is there a possible q-value (satisfying the quota restriction) for which the voter with 7 votes is a dictator? If so, give *an* example of such a q-value. If not, just write no.

yes q=7

(e) Consider the *different* voting system [20; 10, 7, 6, 5, 3, 2, 1]. Is {A, C, E, F, G} a winning coalition? (They have weights 10, 6, 3, 2, and 1 respectively.) If not, explain why not. If so list the critical voters in that coalition.

10+6+3+2+1=22, so yes. Critical are A, C, E.

769
1031
1800
942
2742
981
3723
1277
5000

Problem 3
(5 points) **(a)** The Divided Conglomerate of Terra Firma is sending 50 tax collectors to its 5 capitol cities. Jefferson's method is used. Complete the table:

Principality	Alexandria	Babylon	Carthage	Dothan	Ecbatana	Total
Population	769	1031	942	981	1277	5000
No. of tax collecters: 50			Standard divisor: 100			
Exact Quota	7.69	10.31	9.42	9.81	12.77	XXXXX
Rounded Quota	7	10	9	9	12	47

(3 points) **(b)** Does the first step of Jefferson's method apportion exactly 50 tax collectors? no
If not, should we increase the divisor, or should we decrease the divisor?

(2 points) **(c)** Assume that your new divisor apportions 52 centurions. Should we increase the divisor, or should we decrease it?

Problem 4. **(a)** Michael and Christy just married. Their wedding cake cost them \$360; half is vanilla, and half is chocolate. Michael likes vanilla three times as much as chocolate, and Christy likes chocolate twice as much as vanilla.
(3 points) **(i)** **True** or **False** In Michael's view, the chocolate half is worth \$90 and the vanilla, \$270, and in Christy's view, the chocolate half is worth \$240 and the vanilla \$120.

M
V 270
C 90

C
V 120
C 240

(3 points) **(ii)** Their guest, Jackson, wanted two pieces of cake. Since he is the groom's brother, he got his wish. He took a piece from the vanilla side which is worth \$3 in Michael's view and a piece from the chocolate which is worth \$4 in Christy's view. If Jackson likes each flavor equally, how much are his two pieces worth in his own view? (You may leave your answer in terms of fractions.)

Vanilla: $\boxed{\frac{3}{270} \times 180}$ chocolate: $\boxed{\frac{4}{240} \times 180}$

\$2 \$3

(2 points) **(b)** **True** or **False** In general, the cake-cutting method always produces an envy-free division.

(2 points) **(c)** **True** or **False** In general, a division (not necessarily cake) is called fair if everyone thinks that no one else got more than they deserve.

A B C D E F
4 3 2 1

Problem 5 (2 points each)
Amy, Bob, Chelsea, Domonic, Evan, and Filipe split a pizza using the claim and challenge method. Filipe got a piece in the 1^{st} round. After A claimed a piece in the 2^{nd} round, each person challenged the person before them. In the 3^{rd} round, B passed and C challenged and received a piece. In round 4 the claimant, (the person who started the round), received a piece that round.

(a) At least how many people challenged during the 1^{st} round?

$\boxed{\text{at least 1}}$ (namely F. At most 5.)

(b) Who got a piece in the 2^{nd} round?

E

(c) List all the people who passed in the 3^{rd} round.

B, D

(d) Who was left in the 5^{th} round?

B, D

Problem 6 A die is weighted according to the following probability assignment:

face of die	1	2	3	4	5	6
probability	.2	.1	.15	.1	.15	.3

(1 point) **(a)** The die is rolled once. What is the probability that a 2 is rolled?

.1

(2 points) **(b)** The die is rolled once. Consider the event L, the number on the die is larger than 2. Find Pr(L), the probability of L.

.7

(2 points) **(c)** The die is rolled twice. What is the probability that the sum of the two rolls is is 6?

$.2 \times .15 + .1 \times .1 + .15 \times .15 + .1 \times .1 + .2 \times .15$

Problem 7

A new deck, called Arithmedeck, is formed by using the numbers ranks (2 through 10) and adding the ranks of 0 and 1. (So this deck has *eleven* ranks: 0's, 1's, ..., and 10's. The four usual suits are used: hearts, diamonds, clubs and spades.) **A 6-card hand is used** throughout this problem.

(1 point) **(a)** How many *cards* are in Arithmedeck?

44

(2 points) **(b)** A hand is called *regular* if it does not have the ranks of 0 or 1 in it. How many *regular* hands are there? (Remember, a 6 card hand is used throughout this problem.)

$_{36}C_6$

(2 points) **(c)** A hand is called *sum-deficient* if it contains only the ranks of 0 and 1. How many *sum-deficient* hands are there?

$_{8}C_6$

(1 point) **(d)** How many hands are either *regular* or *sum-deficient*?

$_{36}C_6 + {}_{8}C_6$

(1 point) **(e)** How many hands are not *regular*?

$_{44}C_6 - {}_{36}C_6$

(3 points) **(f)** What is the probability of a 2-pair? (Here is an example of a 2-pair: 8 of hearts, 8 of clubs, 1 of hearts, 1 of diamonds, 10 of spades and 0 of diamonds.)

$$\frac{_{11}C_2 \times {}_4C_2 \times {}_4C_2 \times {}_9C_2 \times {}_4C_1 \times {}_4C_1}{_{44}C_6}$$

Problem 8 Joey is thinking of making an 8 letter password using the 26 lowercase letters of the alphabet.
(2 points) **(a)** How many options does he have?

$$26^8$$

(3 points) **(b)** If he does not want any repeated letters, how many options does he have?

$$26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19$$

(3 points) **(c)** How many passwords contain at least one repeated letter?

$$26^8 - (\text{answer to (b)})$$

(2 points) **(d)** How many passwords are either entirely vowels (like eiaoyiou) or entirely consonants (like mtmkdlsf). (Say there are 6 vowels, including y and 20 consonants).

$$6^8 + 20^8$$

Joey's friend Martin tries to guess the password Joey picks.

(2 points) **(e)** What is the probability that Martin gets the first letter correct?

$$\frac{1}{26}$$

(3 points) **(f)** What is the probability he will get at least 7 letters correct?

$${}_8C_7 \times \left(\frac{1}{26}\right)^7 \times \left(\frac{25}{26}\right)^1 + {}_8C_8 \times \left(\frac{1}{26}\right)^8 \times \left(\frac{25}{26}\right)^0$$

Problem 9 A poll was taken to find out how many clubs college students are members of.
(4 points) **(a)** Fill out the table.

Number of clubs	0	1	2	3	4	5
frequency	30	20	14	11	12	13
cumulative freq.	30	50	64	75	87	100

(10 points) **(b)** Calculate the five number summary.

Min = 0
Q1 = 0
Med = 1.5
Q3 = 3.5
Max = 5

(6 points) **(c)** Write down the mean. (You need not compute it.)

$$\frac{0\times 30 + 1\times 20 + 2\times 14 + 3\times 11 + 4\times 12 + 5\times 13}{100}$$

Problem 10 (10 points) Each year at BU, a certain runner recorded the number of minutes it took him to run a mile. He ran it in 11 minutes the first year, the next year in 9 minutes, the next year in 6 minutes and his last year, 6 minutes again. Compute the standard deviation of this data set. (Give a numerical value for the variance and write the standard deviation in terms of the variance.)

$$\mu = \frac{11+9+6+6}{4} = \frac{32}{4} = 8$$

$$\text{variance} = \frac{(11-8)^2 + (9-8)^2 + (6-8)^2 + (6-8)^2}{4} = \frac{9+1+4+4}{4} = \frac{18}{4}$$

$$= 4.5$$

so $\boxed{\sigma = \sqrt{4.5}}$

Problem 11 The height of the sixth grade boys at East Middle School is approximately normally distributed with mean $\mu = 60$ inches and standard deviation $\sigma = 3$ inches.
(4 points each)

(a) About what percentage of those boys are shorter than 63 inches?

84 %

(b) 16% of them are shorter than about what height? (i.e. What is the 16^{th} percentile?)

57 inches

(c) About what percent are taller than 66 inches?

2.5 %

(d) About what percent is between 51 and 69 inches?

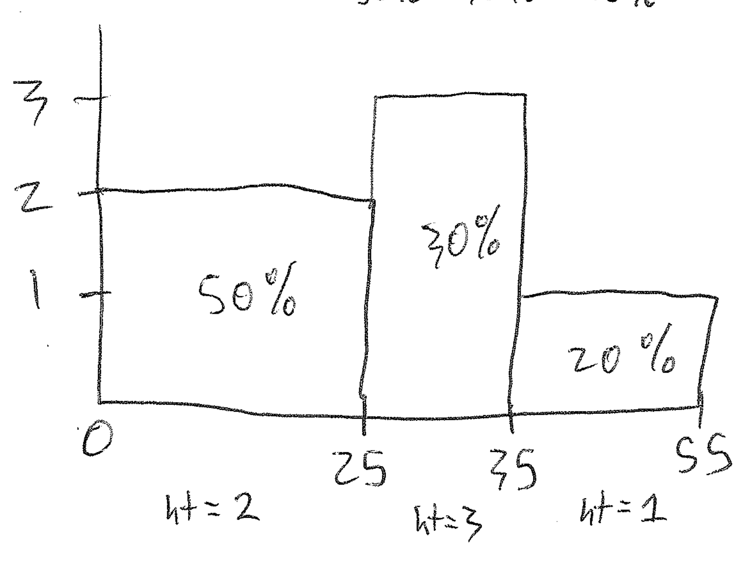
99.7 %

(e) Estimate the 25^{th} percentile height. (You do not need to compute the decimal.)

$60 - .675 \times 3$

Problem 12 (20 points) Lompoc Community College has 300 new students registered for the Fall 2013 semester. Create a histogram of their ages:

ages	[0, 25)	[25, 35)	[35, 55]
frequency	150	90	60
	50%	30%	20%



Problem 13 The Mos Eisley Midi-chlorian Club wanted to estimate the size of the Bantha population on Tatooine. They captured, tagged and released 600 Banthas. A year later, they captured 300 Banthas of which 30 were tagged.

(4 points) **(a)** Find $\hat{p}$, which is the estimate for the fraction of the Bantha population tagged.

$$\hat{p} = \frac{30}{300} = \frac{1}{10}$$

(4 points) **(b)** Find the central estimate.

$$\frac{1}{10} = \frac{600}{N} \Rightarrow N = 6000$$

(4 points) **(c)** Give a formula for the standard error.

$$\frac{\sigma}{n} \approx \sqrt{\frac{\frac{1}{10} \times \frac{9}{10}}{300}}$$

(8 points) **(d)** Assume the standard error is .02. Find the 68% confidence interval for the population of Banthas. You may leave the inequality/interval as fractions.

$$\frac{1}{10} - .02 \le \frac{600}{N} \le \frac{1}{10} + .02$$

50

$$\frac{600}{\frac{1}{10} - .02} \ge N \ge \frac{600}{\frac{1}{10} + .02}$$

Problem 14 (2 points each)

(a) True or **False** ${}_7C_2 < {}_5P_2$ $21 < 20$

(b) True or **False** Five wolves splitting a dead deer is an example of a continuous division problem. I guess?

(Continued on the next page.)

$4! = 24 > 20$

(c) True or **False** In an election with 4 candidates, there are less than 20 possible preference ballots. (i.e. a voter has less than 20 options when making a preference ballot.)

(d) True or **False** The weighted voting system [4; 1, 1, 1, 1, 1, 1, 1] has at least ${}_7C_4$ winning coalitions.

(e) True or **False** In any election in which the borda count and plurality methods produce the same winner, the majority criterion is not violated.

(f) True or **False** The borda count voting method violates the monotonicity fairness criterion.

(g) True or **False** Pretend Theophilis, Pavlos and Sosthenes have equal rights to the inheritance they are splitting using the sealed bids method. If Theophilis and Sosthenes receive all the items, then Pavlos will receive some cash.

(h) True or **False** If the winner of an election does not receive the majority of 1^{st} place votes, the majority criterion is violated.

(i) True or **False** If an election has only two candidates, the pairwise comparison and plurality with elimination methods might produce different winners.

(j) True or **False** When using Hamilton's method, a modified divisor is never used.