Test 3 (form A)

Math in action

Spring 2013

Friday, April 12th

Name (printed): Solutions

Signature: _____

Section number: _____

Directions:

One hour long. No phone, calculator, notes, neighbors, etc...only a pen or pencil is allowed. Absolutely no cheating!

Read carefully. Show your work. Check your work.

Do not turn the page until the professor and/or TA's say so.

Do not write below this line.

	Points
1	
2	
3	
4	
5	

Total =

The exam is out of 100.

For all problems, you may leave your answer in terms of products and sums of numbers, fractions, factorials, ${}_nP_k$, ${}_nC_k$ etc. (Each part of problems 1 through 4 is worth 4 points. Each True/False question is worth 2 points.)

Problem 1 Three friends-Jason, Jenny and Jakshylyk-go to the Fair Share Pizza Parlor. Since they don't like sharing, they each decide to buy their own pizza. The Pizza Parlor sells pizzas in three sizes: medium, large and extralarge. Each pizza has any one of the three sauces: tomato, alfredo and barbecue. The available toppings are as follows: 5 different meat toppings (sausage, roasted chicken...) and 7 different vegetarian toppings (pineapple, olives, onions...).

(a) How many different toppings does the Pizza Parlor have?

$5+7=12$

(b) Jason likes all the toppings and orders a pizza (any size, any sauce) with exactly 1 topping. How many options does he have?

$3 \times 3 \times 12$ (size, sauce, topping)

(c) Jenny likes all toppings too and decides to order a pizza (any size, any sauce) so that one topping is a meat and one is a vegetarian topping. How many options does she have?

$3 \times 3 \times 5 \times 7$

(d) Jakshylyk loves only meat. He decides to order a pizza (any size, any sauce) with exactly 3 meat toppings (and no other toppings). How many options does he have?

$3 \times 3 \times {}_5C_3$

While at the Pizza Parlor, they decide to play poker, using a new deck of cards, called 2-through-queen; this deck is formed by using only the 11 ranks: 2's, 3's, 4's, ... 10's, Jacks, and Queens. (All four suits are still used. So there are $11 \times 4 = 44$ cards in this deck.)

(e) How many hands are in this deck? (A regular, five card hand, is used.)

${}_{44}C_5$

(f) Using this deck, what is the **probability** that a hand is a *three-of-a-kind*? Here is an example of a *three-of-a-kind*: 4 of hearts, 4 of clubs, 4 of diamonds, 7 of spades, and Jack of diamonds.

$$\left({}_{11}C_1 \times {}_4C_3 \times {}_{10}C_2 \times {}_4C_1 \times {}_4C_1 \right) / {}_{44}C_5$$

(rank of triple; suits of triple; rank of other two; suits of other two)

Problem 2 Lilo teaches, at an elementary school, a class of 5th graders consisting of 14 boys and 16 girls. For each of the stated situations, *how many options* does the teacher have on who to give the following gifts? (Note that students who get gifts one week are allowed to continue getting gifts. For example, the student who gets the balloon in (a) may still be one of the four students to receive a gift card in (b). etc.)

(a) One week she (Lilo) decides to give one of her students a balloon.

30

(b) She then gives away four gift cards, worth \$20, \$15, \$10 and \$5 respectively.

${}_{30}P_4$ (order matters)

(c) Then, she gives away two MilkyWay candy bars (each the same size).

~~${}_{30}P_2$~~ ${}_{30}C_2$

(d) It just so happened that both people who got the candy bars were girls, and so the boys complained. The following week, she gives away two MilkyWay's, so that not both go to girls (i.e. at least one boy gets a candy bar).

${}_{30}C_2 - {}_{16}C_2$

(e) After this, she has two \$5 gift cards, and gives exactly one to a boy and one to a girl.

14×16

(f) On the last day of school, she gives away 6 miscellaneous objects. First, she gives away a clock; to a 2nd student, she gives a jacket, to a 3rd student - a typewriter, to the 4th - a cake, to the 5th - a single scoop of ice cream, and to the 6th - an unbreakable lollipop.

$30 \times 29 \times 28 \times 27 \times 26 \times 25$

Problem 3 Zoe has a password, which consists of a sequence of 220 letters. But she uses only the letters A, C, G, and T. (The beginning of her sequence is ACGGTCATAGCT...) Without knowing this sequence, Charles tries to guess it. (He writes down some random sequence 220 letters long using these same four letters.)

What is the probability that...

(a) ...Charles guesses the first letter correctly?

$$\frac{1}{4}$$

(b) ...he guesses the first 3 letters correctly?

$$\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$$

(c) ...each letter in his entire guess is correct?

$$\left(\frac{1}{4}\right)^{220}$$

(d) ...exactly 217 of his letters are correct?

$$_{220}C_{217} \times \left(\frac{1}{4}\right)^{217} \times \left(\frac{3}{4}\right)^{3}$$

(e) ...at least 217 of his letters are correct?

$$_{220}C_{217} \times \left(\frac{1}{4}\right)^{217} \times \left(\frac{3}{4}\right)^{3} + {}_{220}C_{218} \times \left(\frac{1}{4}\right)^{218} \times \left(\frac{3}{4}\right)^{2} +$$
$$_{220}C_{219} \times \left(\frac{1}{4}\right)^{219} \times \left(\frac{3}{4}\right)^{1} + {}_{220}C_{220} \times \left(\frac{1}{4}\right)^{220} \times \left(\frac{3}{4}\right)^{0}$$

(f) ...less than 217 of his letters are correct?

$$1 - (\text{answer to part (e)})$$

Problem 4 Kolmogorov rolls two fair dice. One is a red, 3-sided die, and the other is a blue, 4-sided die.

(a) How many possible outcomes are in the sample space?

$3 \times 4 = 12$

(b) Consider the event R: the number coming up on the red die is greater than the number coming up on the blue die. List all the outcomes in R.

$(2,1), (3,1), (3,2)$

(c) Find Pr(R), the probability of R.

$\frac{3}{12} \left(= \frac{1}{4}\right)$

(d) Consider the event F: the sum of the dice is five and the event S: the sum of the dice is six. Circle the correct answer: $F = \{(1,4),(2,3),(3,2)\}$ $S = \{(2,4),(3,3)\}$

(i) Pr(F) > Pr(S)

(ii) Pr(F) = Pr(S)

(iii) Pr(F) < Pr(S)

Problem 5

(a) **True** or **False** $3! = 6$

(b) **True** or **False** ${}_{100}C_{10} = {}_{100}C_{90}$

(c) **True** or **False** ${}_{10}C_2 > 80$ $\frac{10 \cdot 9}{2} = \frac{90}{2} = 45$

(d) **True** or **False** ${}_{12}P_1 = 4!$

(e) **True** or **False** ${}_8C_4 = {}_8P_4/8!$

(f) **True** or **False** ${}_6P_5 = 6!$