(25 points)
Problem 1
Someone decides to create a new deck of cards, called a wacky-deck. This deck has all 13 ranks and all 4 suits of a normal deck, plus 2 new ranks, circles and stars, for a total of 15 ranks. (Thus we have 15 x 4 = 60 cards). For all parts, you draw a 5-card hand (a hand is an unordered selection of cards).

(5 points)
(a) How many hands are possible?

$${}_{60}C_5$$

(5 points)
(b) How many three-of-a-kind hands are possible? (A three-of-a-kind hand has 3 cards of the same rank, and the other 2 cards are each of a different rank)?

$${}_{15}C_1 \times {}_4C_3 \times {}_{14}C_2 \times ({}_4C_1)^2$$

(5 points)
(c) How many flush hands are possible? (A flush is a hand with all 5 cards in the same suit. In this question, straight-flushes are included as flushes.)

$${}_4C_1 \times {}_{15}C_5$$

(10 points)
(d) What is the probability of drawing either a three-of-a-kind or a flush hand from the wacky-deck?

$$\frac{\text{answer to (b)} + \text{answer to (c)}}{\text{answer to (a)}}$$

(20 points)
Problem 2
Godzilla is trying to decide on a password for his new blackberry phone. He decides he will use a random 8-character password. The characters he can use are the lower case letters a-z, the upper case letters A-Z, and the digits 0-9 (note there are 26 letters in the alphabet).
(2 points)
(a) How many passwords can he make using any combination of lower case letters, upper case letters, and digits?

$$62^8$$

(3 points)
(b) How many passwords begin with 1 lowercase letter and end with two digits?

$$26 \times 62 \times 62 \times 62 \times 62 \times 62 \times 10 \times 10$$

(5 points)
(c) How many passwords contain no digits?

$$52^8$$

(5 points)
(d) How many passwords contain more than 1 digit?

$$\underbrace{62^8}_{\text{all}} - \underbrace{52^8}_{\text{no digits}} - \underbrace{8 \times 52^7 \times 10^1}_{\text{exactly 1 digit}}$$

(5 points)
(e) How many passwords contain the word "moths" (examples include moths4T3 and fT-moths5).

$$4 \times 62^3$$

(25 points: 5 each)

Problem 3

(I) Consider the random experiment of rolling a fair die (numbered 1 through 8) and drawing a card from a standard 52 card deck.

(Ia) Consider the event A: the number on the die is a 2 and the card drawn is red. What is the probability of A?

$$\frac{1}{8} \times \frac{1}{2}$$

(Ib) Consider the event B: the number on the die is odd and the card drawn is a 7. What is the probability of B?

$$\frac{1}{2} \times \frac{1}{13}$$

(II) Now consider the random experiment of rolling a die and drawing a card, but now we use a weighted die. Rolling a 1, 2, 3, 4, or 5 each have a probability of .05, and rolling a six has a probability of .75. We still use the standard poker deck.

(IIa) Consider the event C: the number on the die is odd, and the card is not a J, Q, or K. What is the probability of C?

$$.15 \times \frac{10}{13}$$

(IIb) Consider the event D: the number on the die is a 2 or a 6, and the card is a J. What is the probability of D?

$$.8 \times \frac{1}{13}$$

(IIc) What is the probability of C and D occurring at the same time?

$$0$$

(20 points)

Problem 4

Mark Zuckerberg likes to go out in public disguised as random math professors. His disguises aren't very good, so he gets recognized 66 % of the time (consider him being recognized as a failure). One month, he decides to go out every day (30 days).

(5 points)

(a) What is the probability that he does not get recognized at all?

$${}_{30}C_{30} \times (.37)^{30} \times (.66)^{0}$$

(5 points)

(b) What is the probability that he gets recognized exactly 29 times?

$${}_{30}C_{1} \times (.37)^{1} \times (.66)^{29}$$

(5 points)

(c) What is the probability that he gets recognized more than 28 times?

$${}_{30}C_{1} \times (.37)^{1} \times (.66)^{29} + {}_{30}C_{0} \times (.37)^{0} \times (.66)^{30}$$

(5 points)

(d) What is the probability that he gets recognized 2 or fewer times?

$${}_{30}C_{28} \times (.37)^{28} \times (.66)^{2} + {}_{30}C_{29} \times (.37)^{29} \times (.66)^{1} + {}_{30}C_{30} \times (.37)^{30} \times (.66)^{0}$$

(10 points: 2 each)

Problem 5

True or **False** In a combination, the order does not matter.

True or **False** Suppose you have a random experiment consisting of flipping a coin 500 times. When you add up the probabilities of everything in the sample space, it will add up to less than one.

True or **False** ${}_nC_n = 1$ for every $n > 0$.

True or **False** It is raining and it is cold is an example of disjoint events.

True or **False** You may apply the sum principle when a task can be broken down into a sequence of independent steps.