Math 130 Test 3 (form B) November 4, 2011

Print your name and your section number and sign below, and read the instructions. Do not open the test until you are told to do so.

Name (printed):

Section:

Solutions

Signature:

This test has 5 questions on 5 pages. The total number of points is 150.

When the proctor says you may begin then check that you have a complete test.

Put all your answers in the spaces provided on these sheets. The backs of the test sheets are blank and may be used for scratch work. More scratch paper is available on request.

You must show all your work. You must show enough work to indicate how you got your answer. You will lose credit for incorrect statements or incorrect mathematical expressions. Neatness and clarity are important. You will lose credit if we cannot decipher your answer.

You will be graded on what you write in the space provided for your work. Cross out any scratch work, or label it as scratch. If your work is not in the space provided, indicate clearly where we may find it, and label it. Do not give two or more answers for the same problem.

Do not write inside this box.

1	
2	
3	
4	
5	

21

1. (35 points) **Counting I**

The Italian alphabet consists of every letter of the English alphabet *except for* J, K, W, X and Y. An Italian company is issuing 6-character long passwords to its employees, using letters from the Italian alphabet and digits 1-5.

(1) (5 points) How many passwords are possible?

21^6

(2) (10 points) How many passwords contain only digits?

5^6

(3) (10 points) How many passwords begin with three letters and end with one letter?

$21 \times 21 \times 21 \times 26 \times 26 \times 21$

(4) (10 points) How many passwords contain fewer than five digits?

$21^6 - \left({}_6C_5 \times 5^5 \times 21^1 + {}_6C_6 \times 5^6 \times 21^0 \right)$

2. (40 points) **Counting II**
Tori and PJ are playing poker with a modified deck of cards: They have added an extra denomination, called "Princess." There is one Princess in each suits. As with regular poker, the order that the cards are in doesn't matter and hands consist of 5 cards. They call their version of poker "Princess Poker."

(1) (5 points) How many cards are in the Princess Poker deck?

56

(2) (5 points) How many Princess Poker hands are possible?

$_{56}C_5$

(3) (10 points) How many two pair Princess Poker hands are possible?

$_{14}C_2 \times {}_4C_2 \times {}_4C_2 \times {}_{48}C_1$

ranks of pairs | suits of pairs | extra card

(4) (10 points) How many four-of-a-kind Princess Poker hands are possible?

$_{14}C_1 \times {}_4C_4 \times {}_{52}C_1$

rank of 4-of-a-kind | suits (all) | extra card

(5) (10 points) How many Princess Poker hands are there in which all cards belong to the same suit?

$_4C_1 \times {}_{14}C_5$

suit | ranks

3. (25 points) **Probability I**

what?

Sealed Bids (5 points each) Suppose we have a 20-card deck of cards containing the 2, 3, 4, 5, and 6 of each suit. Consider an experiment: **Draw a card from the 20-card deck, roll a fair 12-sided die, flip a fair coin.**

(1) What is the size of the sample space for this experiment?

$20 \times 12 \times 2 \quad (= 480)$

(2) Consider the event E: Draw a red card, roll a multiple of 4, flip tails. How many outcomes are in the event E?

$10 \times 3 \times 1 \quad (= 30)$

10: red card; 3: 4, 8, 12 on die; 1: tails

(3) What is the probability of the event E?

$$\frac{10 \times 3 \times 1}{20 \times 12 \times 2}$$

(4) Consider the event F: The denomination of the card drawn is different from the number of the die roll. How many outcomes are in the event F?

$20 \times 12 \times 2 - (4 \times 5 \times 2)$

~~since the card can't be same as the card~~

ways the card can equal the die

(5) What is the probability of the event F?

$$\frac{20 \times 12 \times 2 - (4 \times 5 \times 2)}{20 \times 12 \times 2}$$

what?

pr(S) = .81 pr(F) = .19

4. (30 points) **Claim-and-Challenge** (10 points each)
Kara, a college basketball player, is shooting 8 free throws. She makes 81% of her free throws.

(1) What is the probability that Kara will make all 8 free throws?

$${}_8C_8 \times (.81)^8 \times (.19)^0$$

(2) What is the probability that Kara will make 3 free throws out of 8?

$${}_8C_3 \times (.81)^3 \times (.19)^5$$

(3) What is the probability that Kara will make at least 6 free throws out of 8?

6, 7, or 8:

$${}_8C_6 \times (.81)^6 \times (.19)^2 + {}_8C_7 \times (.81)^7 \times (.19)^1 + {}_8C_8 \times (.81)^8 \times (.19)^0$$

5. (20 points) **True or False?** (4 points each)
Circle "True" at each statement that is always true, and circle "False" at each statement is not always true.

(a) True / False $6! = {}_6P_6$.

(b) True / False Whenever the sample space of a random experiment has n outcomes and an event E in the sample space has k outcomes, then $P(E) = k/n$.

(c) True / False Whenever a counting task is broken into steps, the number of ways to complete the counting task is the product of the number of ways to complete each step. (independent steps)

(d) True / False If a coin is doctored so that flipping heads is four times as likely as flipping tails, then the probability of flipping heads is 0.75.

(e) True / False ${}_nC_n = {}_nP_0$