Section 1.4. : One Person – Multiple Votes; Multiple alternatives

But first...

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Here's the algebra behind the UN example last time.

Recall:

Example

For **substantive matters**, need:

- "Yes" from all 5 permanent members, plus
- at least 4 non-permanent members.

Want ($\color{red}{\text{10 non-pmt}}$) + ($\color{blue}{\text{4 pmt}}$) < q,

("Not enough")

Want $(\textcolor{red}{10 \text{ non-pmt}}) + (\textcolor{blue}{4 \text{ pmt}}) < q$,

("Not enough")

but also $(\textcolor{red}{4 \text{ non-pmt}}) + (\textcolor{blue}{5 \text{ pmt}}) = q$.

("Enough")

Let $X =$ (# of permanent member "yes" votes) and

$Y =$ (# of non-permanent member "yes" votes).

Let $X =$ (# of permanent member "yes" votes) and

$Y =$ (# of non-permanent member "yes" votes).

So $10Y + 4X < q$
and $4Y + 5X = q$.

$10Y + 4X < q$ and $4Y + 5X = q$ imply:

$10Y + 4X < 4Y + 5X$

$10Y + 4X < q$ and $4Y + 5X = q$ imply:

$10Y + 4X < 4Y + 5X$

So $6Y < X$.

We just need $6Y < X$, so...

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Might as well use $Y = 1$ and $X = 7$.

We just need $6Y < X$, so...

Might as well use $Y = 1$ and $X = 7$.

Then $q = 4Y + 5X =$
$(4 \times 1) + (5 \times 7) = 39$.

That where we get

[39 : 7, 7, 7, 7, 7, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

Back to Section 1.4. : One Person –
Multiple Votes; Multiple alternatives

- Multiple votes per voter with multiple candidates gets pretty complicated.

- Multiple votes per voter with multiple candidates gets pretty complicated.
- We'll just do an extended example.

Example

The Academic Board at Clearview State College consists of the deans of the 6 schools: Science, Humanities, Engineering, Nursing, Business, and Art.

> **Example**
>
> The Academic Board at Clearview State College consists of the deans of the 6 schools: Science, Humanities, Engineering, Nursing, Business, and Art.
> Each dean controls a number of votes proportional to their school's enrollment.

Example (Cont'd)

This year the weights are:

Science:	22	Engineering:	15
Humanities:	30	Nursing:	5
Business:	19	Art:	9

Example (Cont'd)

- They're electing 5 representatives to the Student Advisory Board.

Example (Cont'd)

- They're electing 5 representatives to the Student Advisory Board.
- With more than two candidates, each dean allocates his/her votes to the various candidates.

Example (Cont'd)

- ▶ Each school presents 3 candidates.

Example (Cont'd)

- Each school presents 3 candidates.
- The deans vote, allocating their votes among the 18 candidates.

Example (Cont'd)

- The top 5 vote candidates are then selected for the board.

Example (Cont'd)

- Note that there are 6 schools but only 5 positions, so some school will lose out.
- Nursing and Art don't have many votes so they might band together.

Seems pretty straightforward…

Seems pretty straightforward... but...

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The Dean of Humanities has some devious plans!

- The Dean of Humanities assumes that others will vote for their own students, except Nursing and Art will band together.

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- He sees a way to get 2 (not just 1) of his own students elected.

- Indeed, Humanities holds more votes (30) than twice Art (9) and Nursing (5) combined.

- Indeed, Humanities holds more votes (30) than twice Art (9) and Nursing (5) combined.
- The Humanities Dean can split his vote evenly between 2 of his own students, and both can still beat the Nursing/Art coalition.

The results of the election are:

S1:	22	**H1**:	15
B1:	19	**H2**:	15
E1:	15	**N1**:	14

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S1:	22	**H1**:	15
B1:	19	**H2**:	15
E1:	15	**N1**:	14

The five new members of the Student Advisory Board are: **S1**, **B1**, **E1**, **H1**, and **H2**.

The Dean of Humanities' plan worked:

The Dean of Humanities' plan worked:

- Most deans voted for one of their own students
- The deans of Art and Nursing joined forces to support one of the nursing students.

How could the others stop this plot?

How could the others stop this plot?

Business and Science don't need all their votes behind one candidate.

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Business and Science don't need all their votes behind one candidate.

They could split some off to help Art/Nursing stop Humanities.

How could the others stop this plot?

Business and Science don't need all their votes behind one candidate.

They could split some off to help Art/Nursing stop Humanities.

As we see, multiple votes per voter with multiple candidates gets pretty complicated.

We'll pretty much ignore it.

Section 1.5. : Breaking Ties

Tie breaking can be tricky.
In real life, to break a tie people often

▸ use a **different voting method**

Tie breaking can be tricky.
In real life, to break a tie people often

- ▸ use a **different voting method**
- ▸ or have a **new election**

Neither of these strategies is guaranteed to break the tie though. Sometimes you just have to flip a coin.

Example

Mr. Jones takes his three children out for ice cream.

Example

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- ▸ The kids don't agree on which ice cream parlor is best.

> **Example**
>
> Mr. Jones takes his three children out for ice cream.
>
> - The kids don't agree on which ice cream parlor is best.
> - He asks them to **rank** the three ice cream parlors in town.

Example (Continued)

Their rankings are:

	Mary	Joe	Ann
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

Now, which place wins?

- Using the Plurality method,

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- ▸ Using the Plurality method, it's a tie (1-1-1).

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Now, which place wins?

- Using the Plurality method, it's a tie (1-1-1).
- Using Borda count, it's a tie (6-6-6).

Now, which place wins?

- Using the Plurality method, it's a tie (1-1-1).
- Using Borda count, it's a tie (6-6-6).
- The other two are also ties.

- All the methods we've studied end up in a **three way tie**!

- **All** the methods we've studied end up in a **three way tie**!
- Just have to flip a coin (or roll a die).

Not all ties are this intractable. Other ties can be broken by either applying a **different method** to the existing preference schedule, or by **holding an additional election**.

Closing remarks on Chapter 1:

> **Remark**
>
> ► *In any election, it should be clear which method is being used. Different methods produce different results.*

Closing remarks on Chapter 1:

Remark

- *In any election, it should be clear which method is being used. Different methods produce different results.*
- *The rules for breaking ties should also be spelled out, including unsolvable ties.*

Next time: Start Chapter 2 – Fair Division and Apportionment.