# Today's plan:

- Section 1.2.4. : Plurality with Elimination Method
  and a second Fairness Criterion:
  The Monotocity Criterion.

**Plurality with Elimination** is a third voting method.

It is more complicated than Plurality but less complicated than Borda Count.

The method can be thought of as a reverse plurality:

- In the **plurality** method the candidate with the most first place votes is the winner.

The method can be thought of as a reverse plurality:

- In the **plurality** method the candidate with the most first place votes is the winner.
- In **plurality with elimination**, the candidate with the fewest first place votes is eliminated.

- In each round, we first check if someone has a majority of first place votes.

- In each round, we first check if someone has a majority of first place votes. If so, this candidate is declared the winner.

- In each round, we first check if someone has a majority of first place votes. If so, this candidate is declared the winner.
- Otherwise, the candidate with the fewest first place votes is eliminated.

- Any first place votes for the eliminated candidate are transferred to the second place choice, and the votes for the second place choice are transferred to third place, etc.

In other words,

- The preference schedule is recompiled by shifting things up to close gaps.

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- The process is repeated, and in each round someone is eliminated.

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- The preference schedule is recompiled by shifting things up to close gaps.
- The process is repeated, and in each round someone is eliminated.
- Eventually, someone will have a majority, and they win.

Example

Find the winner of the Math Club president election using the plurality with elimination method.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 8 | 6 | 1 | 1 | 4 |
| 1st | A | C | C | B | B |
| 2nd | B | D | D | D | C |
| 3rd | C | B | A | C | D |
| 4th | D | A | B | A | A |

Check if anyone has a majority yet....

# Round 1:

- no one has a majority, so...

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- no one has a majority, so...
- we find the candidate with the fewest first place votes.

# Round 1:

- no one has a majority, so...
- we find the candidate with the fewest first place votes. That's **D**, so...

# Round 1:

- no one has a majority, so...
- we find the candidate with the fewest first place votes. That's **D**, so...
- **D** is eliminated.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 8 | 6 | 1 | 1 | 4 |
| 1st | A | C | C | B | B |
| 2nd | B | D | D | D | C |
| 3rd | C | B | A | C | D |
| 4th | D | A | B | A | A |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 8 | 6 | 1 | 1 | 4 |
| 1st | A | C | C | B | B |
| 2nd | B | | | | C |
| 3rd | C | B | A | C | |
| 4th | | A | B | A | A |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 8 | 6 | 1 | 1 | 4 |
| 1st | A | C | C | B | B |
| 2nd | B | ↑ | ↑ | ↑ | C |
| 3rd | C | B | A | C | ↑ |
| 4th | ↑ | A | B | A | A |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 8 | 6 | 1 | 1 | 4 |
| 1st | A | C | C | B | B |
| 2nd | B | B | A | C | C |
| 3rd | C | A | B | A | A |

# Round 2:

- no candidate has a majority

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- no candidate has a majority
- fewest first place votes is candidate

# Round 2:

- no candidate has a majority
- fewest first place votes is candidate **B**.

# Round 2:

- no candidate has a majority
- fewest first place votes is candidate **B**.
- **B** is eliminated.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 8 | 6 | 1 | 1 | 4 |
| 1st | A | C | C | B | B |
| 2nd | B | B | A | C | C |
| 3rd | C | A | B | A | A |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 8 | 6 | 1 | 1 | 4 |
| 1st | A | C | C | | |
| 2nd | | | A | C | C |
| 3rd | C | A | | A | A |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 8 | 6 | 1 | 1 | 4 |
| 1st | A | C | C | ↑ | ↑ |
| 2nd | ↑ | ↑ | A | C | C |
| 3rd | C | A | ↑ | A | A |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 8 | 6 | 1 | 1 | 4 |
| 1st | A | C | C | C | C |
| 2nd | C | A | A | A | A |

Now **C** has a majority of first place votes, and so C wins.

# Remarks:

## Remarks:

- Since D had no first-place votes, we didn't have to re-compute after round 1.

## Remarks:

- Since D had no first-place votes, we didn't have to re-compute after round 1.
- When using plurality with elimination, you can first eliminate anyone with no first place votes.

## Remarks:

- Sometimes a candidate will attain a majority in an early round, and the process can stop.

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- Sometimes a candidate will attain a majority in an early round, and the process can stop.
- When candidates are eliminated, some of the columns might become identical and can be combined.

# Plurality with Elimination Method

- If a candidate has a majority of first place votes, this candidate is declared the winner.
- Otherwise the candidate with the fewest first place votes is eliminated in each round.

- Rounds are repeated with a recompiled preference schedule until someone has a majority.

**Question**

*Does Plurality with Elimination satisfy the Majority criterion?*

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**YES!**

Example

- The site for the 2022 Winter Olympics is being decided among 4 cities, A, B, C, and D, using plurality with elimination.

**Example**

- The site for the 2022 Winter Olympics is being decided among 4 cities, A, B, C, and D, using plurality with elimination.
- There are 87 voters in the committee, so 44 votes constitute a majority.

**Example**

- Before the vote it seems 28 people support A, 16 support B, 17 support C, and 26 support D.

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- Before the vote it seems 28 people support A, 16 support B, 17 support C, and 26 support D.

(Note B and C are close!)

Example

Now 2 of the 17 supporters of C notice that A is way ahead of C, and decide to back the winner. They switch to A.

Example

Now 2 of the 17 supporters of C notice that A is way ahead of C, and decide to back the winner. They switch to A.

Now A has 30, B has 16, C has 15, D has 26.

Example

Now 2 of the 17 supporters of C notice that A is way ahead of C, and decide to back the winner. They switch to A.

Now A has 30, B has 16, C has 15, D has 26.

(That small changed switched "last place" from B to C!)

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | C | B | D |
| 2nd | B | C | B | A | B |
| 3rd | C | D | A | D | A |
| 4th | D | B | D | C | C |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | C | B | D |
| 2nd | B | C | B | A | B |
| 3rd | C | D | A | D | A |
| 4th | D | B | D | C | C |

Find the winner using the Plurality with Elimination method.

**Round 1:** Now C is the city with the fewest first place votes, 15, so it gets eliminated.

| Choice | Number of ballots | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | C | B | D |
| 2nd | B | C | B | A | B |
| 3rd | C | D | A | D | A |
| 4th | D | B | D | C | C |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | | B | D |
| 2nd | B | | B | A | B |
| 3rd | | D | A | D | A |
| 4th | D | B | D | | |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A |  | B | D |
| 2nd | B |  | B | A | B |
| 3rd |  | D | A | D | A |
| 4th | D | B | D |  |  |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | D |
| 2nd | B | D | A | A | B |
| 3rd | D | B | D | D | A |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | D |
| 2nd | B | D | A | A | B |
| 3rd | D | B | D | D | A |

**Round 2:** The first place votes now are A:

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | D |
| 2nd | B | D | A | A | B |
| 3rd | D | B | D | D | A |

**Round 2:** The first place votes now are A: 30, B:

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | D |
| 2nd | B | D | A | A | B |
| 3rd | D | B | D | D | A |

**Round 2:** The first place votes now are A: 30, B: 31, and D:

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | D |
| 2nd | B | D | A | A | B |
| 3rd | D | B | D | D | A |

**Round 2:** The first place votes now are A: 30, B: 31, and D: 26, so D is eliminated.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | D |
| 2nd | B | D | A | A | B |
| 3rd | D | B | D | D | A |

**Round 2:** The first place votes now are A: 30, B: 31, and D: 26, so D is eliminated.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | |
| 2nd | B | | A | A | B |
| 3rd | | B | | | A |

**Round 2:** The first place votes now are A: 30, B: 31, and D: 26, so D is eliminated.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | ↑ |
| 2nd | B | ↑ | A | A | B |
| 3rd | ↑ | B | ↑ | ↑ | A |

**Round 2:** The first place votes now are A: 30, B: 31, and D: 26, so D is eliminated.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | B |
| 2nd | B | B | A | A | A |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | B |
| 2nd | B | B | A | A | A |

- B now has

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | B |
| 2nd | B | B | A | A | A |

- B now has 57 first place votes which is a majority.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | A | B | B | B |
| 2nd | B | B | A | A | A |

- B now has 57 first place votes which is a majority.
- So B is the winner!

# Remarks:

## Remarks:

- It looks like the two people who switched their votes were wrong. A didn't win!

## Remarks:

- It looks like the two people who switched their votes were wrong. A didn't win!
- Actually, something **more serious** than that happened.

What would have happened if they hadn't changed their vote?

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | C | C | B | D |
| 2nd | B | A | B | A | B |
| 3rd | C | D | A | D | A |
| 4th | D | B | D | C | C |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | C | C | B | D |
| 2nd | B | A | B | A | B |
| 3rd | C | D | A | D | A |
| 4th | D | B | D | C | C |

**Round 1:** Here B has the fewest votes, and it is eliminated.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | C | C | B | D |
| 2nd | B | A | B | A | B |
| 3rd | C | D | A | D | A |
| 4th | D | B | D | C | C |

**Round 1:** Here B has the fewest votes, and it is eliminated.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | C | C | | D |
| 2nd | | A | | A | |
| 3rd | C | D | A | D | A |
| 4th | D | | D | C | C |

**Round 1:** Here B has the fewest votes, and it is eliminated.

| Choice | Number of ballots | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | C | C | ↑ | D |
| 2nd | ↑ | A | ↑ | A | ↑ |
| 3rd | C | D | A | D | A |
| 4th | D | ↑ | D | C | C |

**Round 1:** Here B has the fewest votes, and it is eliminated.

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | C | C | A | D |
| 2nd | C | A | A | D | A |
| 3rd | D | D | D | C | C |

| **Choice** | **Number of ballots** | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | C | C | A | D |
| 2nd | C | A | A | D | A |
| 3rd | D | D | D | C | C |

- A has 44 first place votes, a majority.

| Choice | Number of ballots | | | | |
|---|---|---|---|---|---|
| | 28 | 2 | 15 | 16 | 26 |
| 1st | A | C | C | A | D |
| 2nd | C | A | A | D | A |
| 3rd | D | D | D | C | C |

- A has 44 first place votes, a majority.
- A would have been the winner, if those two voters had not changed their vote.

Before those 2 switched, B was in last place and A would have won.

Before those 2 switched, B was in last place and A would have won.

After they switched, B was no longer in last place, and actually ended up winning!

So those two voters actually **took the victory away** from A, by throwing their support **in favor** of A!

So those two voters actually **took the victory away** from A, by throwing their support **in favor** of A!

This is one of the deficiencies of the plurality with elimination method: it violates the **monotonicity criterion**.

# Another Fairness Criterion:

**Monotonicity Criterion**

If there is a change in the preference schedule that favors the winner, then the result of the election ought not to change.

**Remark**

*The Plurality Method and the Borda Count Method satisfy the Monotonicity Criterion.*

*If someone changes their mind to back the winner, that just gives the winner even more votes/points.*

## Variations of the plurality with elimination method

The International Olympic Committee actually uses a variation of the plurality with elimination method, called the Hare method.

Example

- In the **Hare method** voters don't use a preference ballot to rank the candidates.

Example

- In the **Hare method** voters don't use a preference ballot to rank the candidates.
- Instead they vote for a single candidate.

Example

- In the **Hare method** voters don't use a preference ballot to rank the candidates.
- Instead they vote for a single candidate.
- If someone gets a majority, they win.

Example

- Otherwise, the candidate with the fewest votes is eliminated, and a **new** election is held with the remaining ones.

Example

- Otherwise, the candidate with the fewest votes is eliminated, and a **new** election is held with the remaining ones.
- The process is repeated until someone gets a majority.

Example

- Otherwise, the candidate with the fewest votes is eliminated, and a **new** election is held with the remaining ones.
- The process is repeated until someone gets a majority.

(We don't want to deal with multiple elections, so we use preference schedules.)

Example

In politics, some elections use a **runoff election** between the top two candidates, if neither of them got a majority.

Example

In politics, some elections use a **runoff election** between the top two candidates, if neither of them got a majority.
All but the top two candidates are eliminated, instead of one at a time like the Hare method.

To recap, we now have:

Voting Methods: 1)

To recap, we now have:

Voting Methods: 1) Plurality. 2)

To recap, we now have:

Voting Methods: 1) Plurality. 2) Borda Count. 3)

To recap, we now have:

Voting Methods: 1) Plurality. 2) Borda Count. 3) Plurality with Elimination. 4) ???

Fairness Criteria: 1)

To recap, we now have:

Voting Methods: 1) Plurality. 2) Borda Count. 3) Plurality with Elimination. 4) ???

Fairness Criteria: 1) Majority. 2)

To recap, we now have:

Voting Methods: 1) Plurality. 2) Borda Count. 3) Plurality with Elimination. 4) ???

Fairness Criteria: 1) Majority. 2) Monotonicity. 3) ??? 4) ???

Next time:
Section 1.2.5 : Pairwise Comparison Method
and
Section 1.2.6.