Today's plan:

- Section 4.2: Normal Distribution

Characteristics of a data set:

- mean
- median
- standard deviation
- five-number summary

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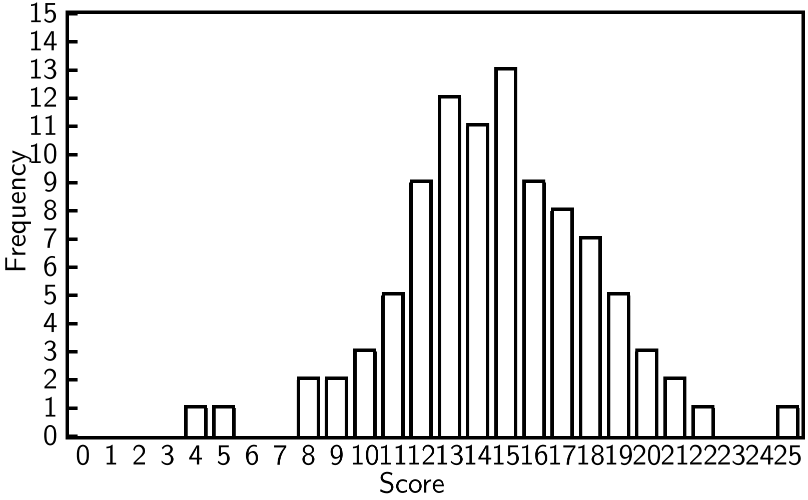
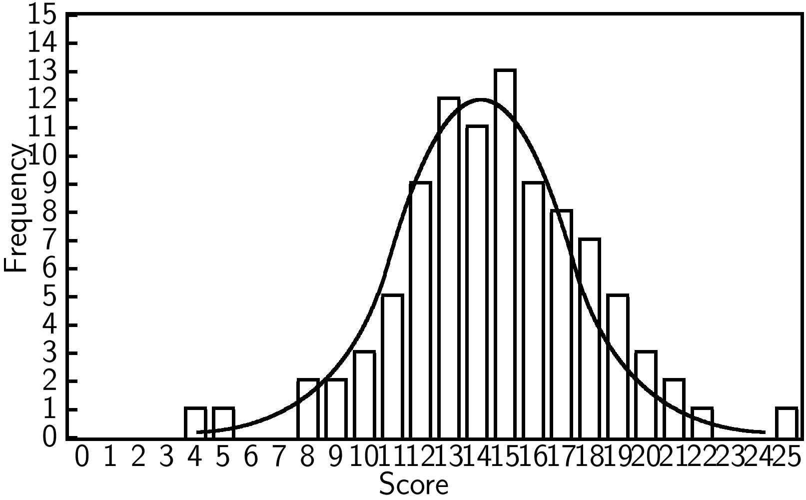
- median
- standard deviation
- five-number summary

These don't always tell you everything though.

Bar graph for scores of Math 109 quiz:


Frequency
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Score



It's somewhat close to a bell-shaped curve.

Definition

- We call that bell-shaped curve a **normal curve**.

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- A data set whose distribution is a normal curve has a **normal distribution**.

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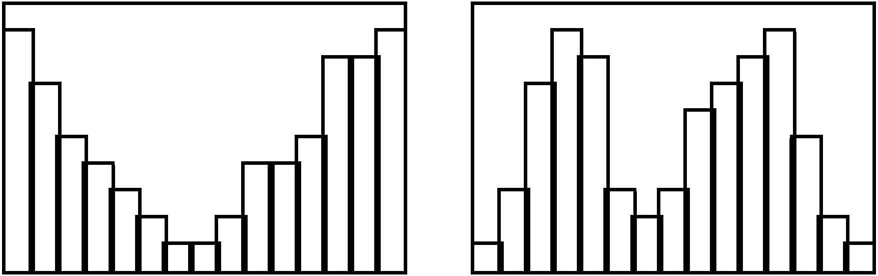
- it's close, but not super close to the normal curve
- Note the outliers
- the spike at 15
- the whole picture is a little skewed to the right

Non-normal distributions:

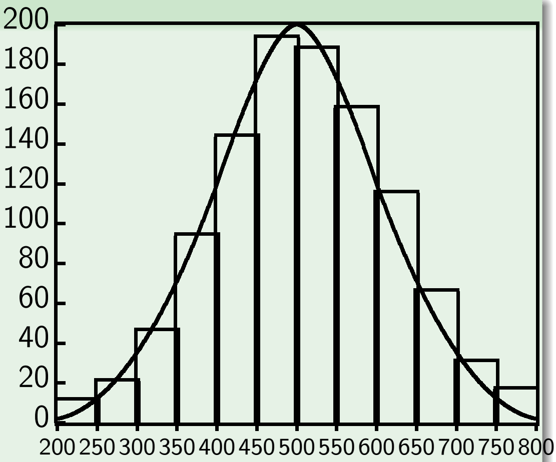


- The first one has an inverted bell shape.
- The second one is called a bimodal distribution.

Lots of data sets do have normal or near-normal distributions though, so we're interested in them.

Example

Score	Freq.
[200, 250)	11,393
[250, 300)	20,874
[300, 350)	46,583
[350, 400)	94,037
[400, 450)	143,630
[450, 500)	193,470
[500, 550)	187,773
[550, 600)	158,138
[600, 650)	115,401
[650, 700)	66,066
[700, 750)	30,503
[750, 800]	16,857



This is a normal distribution. (SAT scores)

Section 4.2.2: Properties of Normal Distributions

- Normal distributions **approximate** real life distributions

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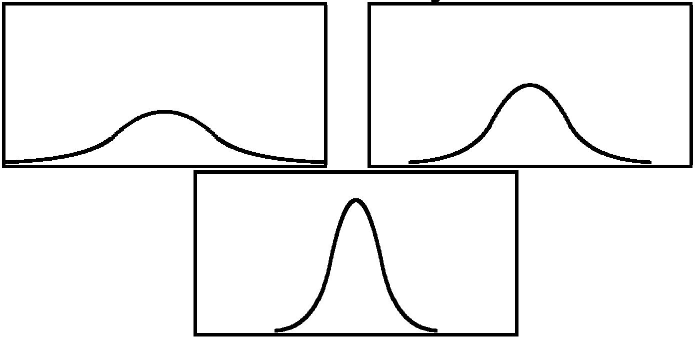
- Knowing the properties of normal curves, we can approximate the properties of near-normal data sets.

Normal curves can vary:



but they all have common properties.

Symmetry A normal curve is symmetric about the vertical line at the mean μ. This is called its **axis of symmetry**.

Concavity A normal curve has a concave up region on the left, a concave down region in the middle, and a concave up region on the right.

Inflection points The points where the curve changes concavity are called inflection points.

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- There are two inflection points, each one standard deviation from the center.

68-95-99.7 Principle Tells where the data points should be:

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- 68% of the data set is located under the concave down part of the curve. That's within σ of μ.
- 95% of the data points are within 2 σ's of μ.
- 99.7% of the data points are within 3 σ's of μ.

Quartiles

$Q1 = \mu - 0.675\sigma$ and

$$Q3 = \mu + 0.675\sigma$$

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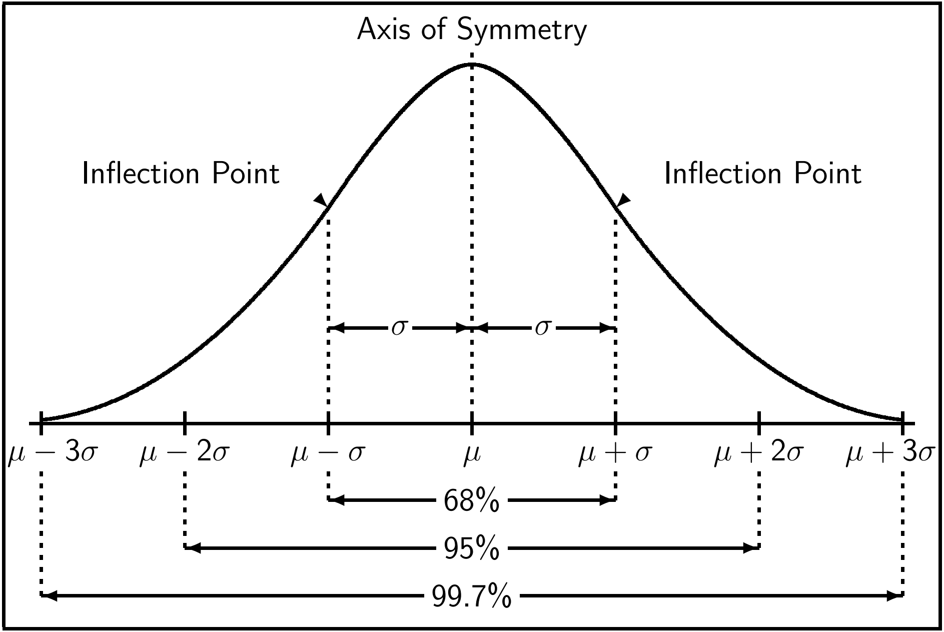
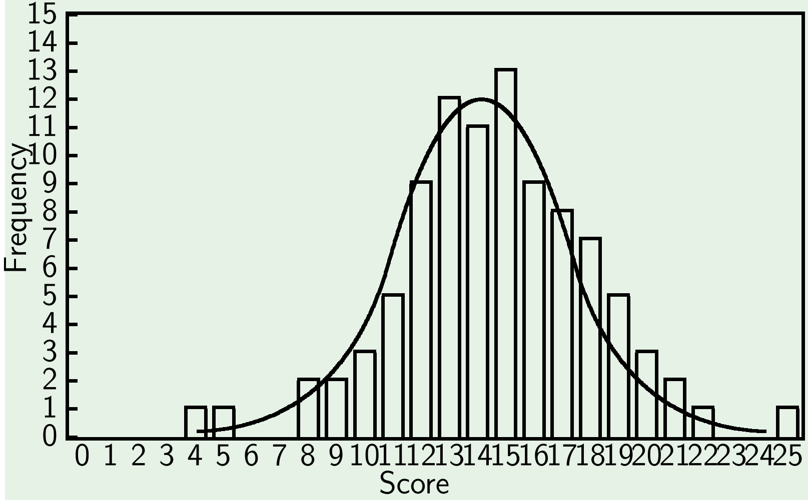
So 50% of data points are within 0.675σ of μ.


Axis of Symmetry
Inflection Point
Inflection Point
σ
σ
$\mu - 3\sigma$
$\mu - 2\sigma$
$\mu - \sigma$
μ
$\mu + \sigma$
$\mu + 2\sigma$
$\mu + 3\sigma$
68%
95%
99.7%

For near-normal distributions, the above properties should be taken as approximations.

Example

Recall Math 109 quiz:



score	4	5	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	25
freq.	1	1	2	2	3	5	9	12	11	13	9	8	7	5	3	2	1	1
cum fr.	1	2	4	6	9	14	23	35	46	59	68	76	83	88	91	93	94	95

We found earlier:

- $\mu = 14.64$

score	4	5	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	25
freq.	1	1	2	2	3	5	9	12	11	13	9	8	7	5	3	2	1	1
cum fr.	1	2	4	6	9	14	23	35	46	59	68	76	83	88	91	93	94	95

We found earlier:

- $\mu = 14.64$
- $\sigma = 3.35$

score	4	5	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	25
freq.	1	1	2	2	3	5	9	12	11	13	9	8	7	5	3	2	1	1
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We found earlier:

- $\mu = 14.64$
- $\sigma = 3.35$
- 62 out of 95 scores are within one σ from μ

$$\frac{62}{95} \cdot 100\% = 65.3\%$$

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We found earlier:

- $\mu = 14.64$
- $\sigma = 3.35$
- 62 out of 95 scores are within one σ from μ

$$\frac{62}{95} \cdot 100\% = 65.3\%$$

which is close to the ideal 68%

- number of scores within 2 σ's of μ:

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  $\mu - 2\sigma = 7.94 \quad \text{and} \quad \mu + 2\sigma = 21.34,$

  so 91 out of 95 (i.e., 95.8%), close to the ideal 95%.

- number of scores within 2 σ's of μ:

  $$\mu - 2\sigma = 7.94 \quad \text{and} \quad \mu + 2\sigma = 21.34,$$

  so 91 out of 95 (i.e., 95.8%), close to the ideal 95%.
- Finally, 93 out of 95 (i.e., 97.9%) points are within 3 σ's of μ.

- number of scores within 2 σ's of μ:

  $$\mu - 2\sigma = 7.94 \quad \text{and} \quad \mu + 2\sigma = 21.34,$$

  so 91 out of 95 (i.e., 95.8%), close to the ideal 95%.
- Finally, 93 out of 95 (i.e., 97.9%) points are within 3 σ's of μ.
  All of this is close to the ideal 68-95-99.7 principle.

We can also estimate the quartiles for the Math 109 quizzes:

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- $Q1 \approx \mu - 0.675\sigma = 12.38$
- $Q3 \approx \mu + 0.675\sigma = 16.90$

(Faster than actually calculating them.)

Conclusion: If you know that the distribution of data points is normal, it is enough to know just 2 numbers to describe the data set completely:

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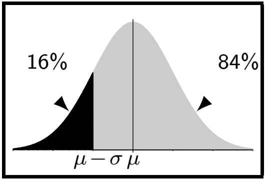
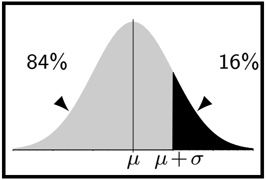
- Mean μ
- Standard deviation σ

Section 4.2.3: Using the 68-95-99.7 Principle

- Each tail outside one σ contains

$$\frac{100\% - 68\%}{2} = 16\%$$





This means that a random point has:

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- 84% chance of being above $\mu - \sigma$

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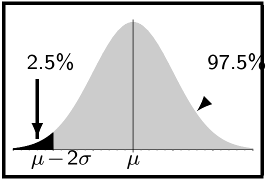
- 84% chance of being above $\mu - \sigma$

$$\mathrm{pr}(x < \mu - \sigma) = 0.16 \quad \text{and}$$

$$\mathrm{pr}(x \geq \mu - \sigma) = 0.84.$$

Similarly, each tail outside 2σ contains

$$\frac{100\% - 95\%}{2} = 2.5\%$$



Example

The weights of 6-month-old baby boys in the U.S. have a near-normal distribution with

- $\mu = 17.25$ lbs
- $\sigma = 2$ lbs

Example

Questions:

- **(a)** What can we say about babies whose weight is 19.25 lb.?

Example

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- **(a)** What can we say about babies whose weight is 19.25 lb.?
- **(b)** If we pick a random baby, what's the probability he's less than 13.25 lb.?

Solution

(a) *The baby whose weight is 19.25 lb.*

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- *is certainly above the average*

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(a) *The baby whose weight is 19.25 lb.*

- *is certainly above the average*
- *Moreover, his weight*

$$19.25 = 17.25 + 2 = \mu + \sigma$$

is precisely one standard deviation above the average.

Solution

Thus

- *84% of babies weigh less than him*

Solution

Thus

- *84% of babies weigh less than him*
- *16% of babies weigh more than him*

We say that his weight is in the 84th **percentile**.

Solution

(b) *A weight of 13.25 lb. or less places a baby at least 2 standard deviations below the average.*

$$13.25 = 17.25 - 2 \cdot 2 = \mu - 2\sigma$$

Solution

(b) *A weight of 13.25 lb. or less places a baby at least 2 standard deviations below the average.*

$$13.25 = 17.25 - 2 \cdot 2 = \mu - 2\sigma$$

The probability for that is 2.5%.

Next Time:

- Section 4.3: Data Collection
- Section 4.3.1: Population v. Sample