# Today's plan:

- Section 2.4.2. (Discrete–different values–equal rights)

Section 2.4.2.: The discrete case of Different Values - Equal Rights: Sealed Bids Method.

**Example**

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- Transform the problem to continuous:
  sell the house and split the money

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- Sell the house for as much as the market is willing to pay.

## Potential problem:

What if the market value is <span style="color:red">lower</span> than the value assigned by both parties? Maybe they don't want to sell the house.

## Solution

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- *They both say yes, so she asks each of them* <span style="color:red">*to make a sealed bid for the house*</span>

*Each party bids what she/he considers to be the value of the house.*

## Solution

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*After the envelopes are opened:*

- *Daira (the highest bid) gets the house*
- *She pays $100K, half the value in her view, to the court*

## Solution

- *The court pays Lew $90K for half the value in his view*

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- *There is a surplus of $10K which is then split in equal parts*
- *The final settlement is: Daira gets the house and pays $95K; Lew gets $95K*

# Remarks:

- ▸ Note that the final settlement is equivalent to averaging/splitting the bids: the house is worth $190K, and each party gets half, namely $95K; he gets it in cash, she gets the house minus $95K.

**Remarks:**

- Note that the final settlement is equivalent to averaging/splitting the bids: the house is worth $190K, and each party gets half, namely $95K; he gets it in cash, she gets the house minus $95K.

- There's a <span style="color:red">cash surplus</span> since the house goes to the highest bidder.

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- ▸ If he bids higher, he may have to pay more than what he considers fair
- ▸ If he bids lower, he may get less money than what he considers his fair share

A house is a typical discrete item.
You can't cut the house into pieces.

**Example**

An estate consisting of a house, a car, and some jewelry is to be divided into equal parts among four heirs. (Even though the jewelry consists of several items, they don't want to split it up.)

The heirs agree to use the method of sealed bids. Here are their bids.

| Item | Taisha | Shakira | Moira | Jennifer |
|------|--------|---------|-------|----------|
| House | 120,000 | 125,000 | 140,000 | 108,000 |
| Car | 40,000 | 48,000 | 46,000 | 35,000 |
| Jewelry | 18,200 | 18,400 | 15,000 | 12,000 |

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Find what each person thinks the whole estate is worth (add down columns), and calculate what each person believes is her fair share (divide her idea of the total by 4).

For each item, highest bid gets it. Comparing opinions of "fair share," they pay each other. Finally, give out any surplus.

| Item | Taisha | Shakira | Moira | Jennifer |
|---|---|---|---|---|
| House | 120,000 | 125,000 | 140,000 | 108,000 |
| Car | 40,000 | 48,000 | 46,000 | 35,000 |
| Jewelry | 18,200 | 18,400 | 15,000 | 12,000 |
| Total | 178,200 | 191,400 | 201,000 | 155,000 |
| Fair share | 44,550 | 47,850 | 50,250 | 38,750 |
| Allocated | 0 | 66,400 | 140,000 | 0 |
| Difference | 44,550 | -18,550 | -89,750 | 38,750 |
| Total surplus = 25,000 | | | | |
| Surplus share | 6,250 | 6,250 | 6,250 | 6,250 |

Final allocation summary

| | Taisha | Shakira | Moira | Jennifer |
|---|---|---|---|---|
| Items | none | Car, Jewelry | House | none |
| Items' Value | 0 | 66,400 | 140,000 | 0 |
| Cash | 50,800 | -12,300 | -83,500 | 45,000 |
| Net total | 50,800 | 54,100 | 56,500 | 45,000 |

# Remarks:

- ▸ Each heir has a different total value of the estate. Therefore, each one has a different value for a fair share.

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- Each heir has a different total value of the estate. Therefore, each one has a different value for a fair share.

- Every heir got more than what she considered a fair share. That's because the winner is always the highest bidder.

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- Jennifer may have bid low trying to get a good deal on the items. However, the net result is that she gets less than the others. It is not to a player's advantage to bid low.

# Sealed Bids Method

▶ Each player submits a bid for each item in a sealed envelope.

▶ Once all the bids are collected, the envelopes are opened.

# Sealed Bids Method

- Each player's bids are added up and divided by the number of players, to obtain the fair share for that player.
- Each item is allocated to the highest bidder.

# Sealed Bids Method

- If the items allocated to a player are worth more than his fair share, he pays the difference.
- If items allocated to a player are worth less than his fair share, he gets the difference.

# Sealed Bids Method

► The surplus cash is divided into equal parts among the players.

End of Chapter 2:
Section 2.5 (short): Different Values
- Different Rights.

In the discrete case (sealed bids), when each fair share is determined, instead of dividing by the number of players,

In the discrete case (sealed bids), when each fair share is determined, instead of dividing by the number of players, find the percentage each player is entitled to.

**Example**

The four owners of The Kleen Car Wash Co. have unanimously decided to break up the company.

**Example**

▶ Each one will get the appropriate percentage of assets:

| A | B | C | D |
|-----|-----|-----|-----|
| 40% | 30% | 20% | 10% |

**Example**

- Each one will get the appropriate percentage of assets:

| A | B | C | D |
|---|---|---|---|
| 40% | 30% | 20% | 10% |

- They use the sealed bids method.

**Example**

The (discrete) assets of the company include: 3 automatic car washers, an auto parts store and a truck.

| Item         | A       | B       | C       | D       |
|--------------|---------|---------|---------|---------|
| Car Washer 1 | 130,000 | 140,000 | 120,000 | 135,000 |
| Car Washer 2 | 150,000 | 145,000 | 155,000 | 120,000 |
| Car Washer 3 | 165,000 | 170,000 | 150,000 | 160,000 |
| Store        | 260,000 | 250,000 | 250,000 | 255,000 |
| Truck        | 45,000  | 40,000  | 30,000  | 35,000  |

| Item | A | B | C | D |
|------|------|------|------|------|
| Car Washer 1 | 130,000 | <u>140,000</u> | 120,000 | 135,000 |
| Car Washer 2 | 150,000 | 145,000 | <u>155,000</u> | 120,000 |
| Car Washer 3 | 165,000 | <u>170,000</u> | 150,000 | 160,000 |
| Store | <u>260,000</u> | 250,000 | 250,000 | 255,000 |
| Truck | <u>45,000</u> | 40,000 | 30,000 | 35,000 |
| Total | 750,000 | 745,000 | 705,000 | 705,000 |
| Company stock | 40% | 30% | 20% | 10% |
| Fair share | 300,000 | 223,500 | 141,000 | 70,500 |
| Allocated | 305,000 | 310,000 | 155,000 | 0 |
| Difference | -5,000 | -86,500 | -14,000 | 70,500 |
| Total surplus = 35,000 | | | | |
| Surplus share | 14,000 | 10,500 | 7,000 | 3,500 |

## Final allocation summary

|              | A            | B           | C         | D      |
|--------------|--------------|-------------|-----------|--------|
| Items        | Store, Truck | Washers 1,3 | Washer 2  | none   |
| Items' value | 305,000      | 310,000     | 155,000   | 0      |
| Cash         | 9,000        | -76,000     | -7,000    | 74,000 |
| Net total    | 314,000      | 234,000     | 148,000   | 74,000 |

**Remarks:** There were two rows where this differs from the equal-rights case:

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**Remarks:** There were two rows where this differs from the equal-rights case:

"Fair share" and "Surplus share."

Don't forget to allocate the surplus at the end, and careful if it's a different-rights problem.

End of Chapter 2.