Chapter 1 exam done. Now back to Chapter 2.

Quick review of Adams'/Webster's/Huntington-Hill's Methods.

Adams' method

Island	**Arisa**	**Beruga**	**Crispa**	**Daria**	**Total**
Pop.	1,205,000	163,000	267,000	165,000	1,800,000
$\mathbf{M} = 18$			$\mathbf{d} = 100{,}000$		
Exact q.	12.050	1.630	2.670	1.650	
Rounded q.	13	2	3	2	20 Too high!

Island	Arisa	Beruga	Crispa	Daria	Total
Pop.	1,205,000	163,000	267,000	165,000	1,800,000
M = 18			**d** = 100,000		
Exact q.	12.050	1.630	2.670	1.650	
Rounded q.	13	2	3	2	20 Too high!

M = 18			**md** = 105,000		
Md Exact q.	11.48	1.55	2.54	1.57	
Rounded q.	12	2	3	2	19 Too high!

Island	Arisa	Beruga	Crispa	Daria	Total
Pop.	1,205,000	163,000	267,000	165,000	1,800,000
M = 18			**d** = 100,000		
Exact q.	12.050	1.630	2.670	1.650	
Rounded q.	13	2	3	2	20 Too high!
M = 18			**md** = 105,000		
Md Exact q.	11.48	1.55	2.54	1.57	
Rounded q.	12	2	3	2	19 Too high!
M = 18			**md** = 110,000		
Md Exact q.	10.95	1.48	2.43	1.5	
Rounded q.	11	2	3	2	18 Just right!

Webster's method

Island	Arisa	Beruga	Crispa	Daria	Total
Pop.	1,205,000	163,000	267,000	165,000	1,800,000
M = 18			**d** = 100,000		
Exact q.	12.050	1.630	2.670	1.650	
Rounded q.	12	2	3	2	19 Too high!

Island	**Arisa**	**Beruga**	**Crispa**	**Daria**	**Total**
Pop.	1,205,000	163,000	267,000	165,000	1,800,000
M = 18			**d** = 100,000		
Exact q.	12.050	1.630	2.670	1.650	
Rounded q.	12	2	3	2	19 Too high!
M = 18			**md** = 110,000		
Md Exact q.	10.955	1.482	2.427	1.500	
Rounded q.	11	1	2	2	16 Too low!

Island	Arisa	Beruga	Crispa	Daria	Total
Pop.	1,205,000	163,000	267,000	165,000	1,800,000
$\mathbf{M} = 18$			$\mathbf{d} = 100{,}000$		
Exact q.	12.050	1.630	2.670	1.650	
Rounded q.	12	2	3	2	19 Too high!
$\mathbf{M} = 18$			$\mathbf{md} = 110{,}000$		
Md Exact q.	10.955	1.482	2.427	1.500	
Rounded q.	11	1	2	2	16 Too low!
$\mathbf{M} = 18$			$\mathbf{md} = 105{,}000$		
Md Exact q.	11.476	1.552	2.543	1.571	
Rounded q.	11	2	3	2	18 Just right!

Huntington-Hill's method

Island	Arisa	Beruga	Crispa	Daria	Total
Pop.	1,205,000	163,000	267,000	165,000	1,800,000
$\mathbf{M} = 18$			$\mathbf{d} = 100{,}000$		
Exact q.	12.050	1.630	2.670	1.650	
Rounded q.	12	2	3	2	19 Too high!

Island	**Arisa**	**Beruga**	**Crispa**	**Daria**	**Total**
Pop.	1,205,000	163,000	267,000	165,000	1,800,000
M = 18				**d** = 100,000	
Exact q.	12.050	1.630	2.670	1.650	
Rounded q.	12	2	3	2	19 Too high!

M = 18				**md** = 105,000	
Md Exact q.	11.476	1.552	2.543	1.571	
Rounded q.	11	2	3	2	18 Just right!

Section 2.3.4: Problems with Apportionment.

We have studied five different apportionment methods: Hamilton's, Jefferson's, Adams', Webster's and Huntington-Hill's.

We have studied five different apportionment methods: Hamilton's, Jefferson's, Adams', Webster's and Huntington-Hill's.
All but one of them (Adams') have been used at some point to apportion the seats of Congress.

Sometimes they agree, sometimes not.

Sometimes they agree, sometimes not.

Which one is the *correct method*?

Sometimes they agree, sometimes not.

Which one is the *correct method*?

Unfortunately, the short answer is: none. They all have drawbacks.

- The standard exact quota is the ideal allocation.

- This is hardly ever an integer, so the next best thing is either the lower quota or the upper quota.

- This is hardly ever an integer, so the next best thing is either the lower quota or the upper quota.
- We expect an apportionment method "should" give each state either its lower quota or its upper quota.

Quota Criterion

An apportionment method ought to allocate to each state either its standard lower quota, or its standard upper quota.

By design, Hamilton's method always respects the quota criterion.

By design, Hamilton's method always respects the quota criterion.

- It gives each state the standard lower quota, plus maybe one surplus seat.

By design, Hamilton's method always respects the quota criterion.

- It gives each state the standard lower quota, plus maybe one surplus seat.
- Those states receiving a surplus seat end up with their standard upper quota.

By design, Hamilton's method always respects the quota criterion.

- It gives each state the standard lower quota, plus maybe one surplus seat.
- Those states receiving a surplus seat end up with their standard upper quota.
- The others get their standard lower quota.

Not all methods respect the quota criterion.

Sometimes in hunting for an *md* that makes everything work, an individual quota will drift past its lower or upper bounds.

Webster's method

Island	**Arisa**	**Beruga**	**Crispa**	**Daria**	**Total**
Pop.	1,205,000	163,000	267,000	165,000	1,800,000
M = 18				**d** = 100,000	
Exact q.	12.050	1.630	2.670	1.650	
Rounded q.	12	2	3	2	19 Too high

M = 18				**md** = 105,000	
Md Exact q.	11.476	1.552	2.543	1.571	
Rounded q.	11	2	3	2	18

- **Webster's method** allocates 11 cabinet seats to Arisa Island, even though its standard exact quota was 12.050.

- **Webster's method** allocates 11 cabinet seats to Arisa Island, even though its standard exact quota was 12.050.
- By the quota criterion it should have gotten either 12 or 13 seats.

- **Webster's method** allocates 11 cabinet seats to Arisa Island, even though its standard exact quota was 12.050.
- By the quota criterion it should have gotten either 12 or 13 seats.
- So Webster's method violates the quota criterion.

- For each of the other methods, examples can be found that violate the quota criterion.

- For each of the other methods, examples can be found that violate the quota criterion.
- However, most examples do respect the quota criterion.

- For each of the other methods, examples can be found that violate the quota criterion.
- However, most examples do respect the quota criterion.
- In fact, of all the examples in this section, only the above example violates the quota criterion. It was designed to do so.

Isn't Hamilton's method then the perfect one?

Isn't Hamilton's method then the perfect one?

Even though Hamilton's method respects the quota criterion, it has several other problems.

Until 1941, every 10 years Congress had to re-decide on the apportionment method and a number of seats.

In 1880, they calculated apportionment tables for House sizes ranging from $M = 275$ to $M = 350$, and something paradoxical appeared.

- When M went up from $M = 299$ and $M = 300$, so a brand new seat was added, you'd expect someone to just gain a seat.

- When M went up from $M = 299$ and $M = 300$, so a brand new seat was added, you'd expect someone to just gain a seat.
- But instead, two states, Illinois and Texas, each gained a seat, and another state, Alabama, lost a seat!

Alabama paradox

An increase in the total number of seats may cause a state to lose one of its seats

Alabama paradox

State	Alabama	Illinois	Texas	Total
Population	1,283,405	3,128,795	1,618,100	50,189,209
House Size: 299		**Std. div.:**	167,856.886	
Exact Q.	7.646	18.640	9.640	
Lower Q.	7	18	9	
Fract.	.646	.640	.640	
Surplus	1			
Total	8	18	9	299
House Size: 300		**Std. div.:**	167,297.363	
Exact Q.	7.671	18.702	9.672	
Lower Q.	7	18	9	
Fract.	.671	.702	.672	
Surplus		1	1	
Total	7	19	10	300

Question

So Hamilton's method has the Alabama paradox, and Jeff/Adams/Web/HH violate the quota criterion. Is there another method out there that satisfies the quota criterion without producing paradoxes?

The Census Bureau people and Congress struggled with this question for nearly two centuries.

The Census Bureau people and Congress struggled with this question for nearly two centuries. The answer turned out to be "no" in 1980.

Two mathematicians, Balinski and Young, proved

Theorem

It is impossible to design an apportionment method that satisfies the quota criterion and avoids the paradoxes.

Two mathematicians, Balinski and Young, proved

Theorem

It is impossible to design an apportionment method that satisfies the quota criterion and avoids the paradoxes.

There is no perfect apportionment method.

Among all the imperfect methods, the Huntington-Hill method is considered by many experts to be the most reasonable one.

Next time:

Section 2.4.: Different values - Equal rights
Section 2.4.1: The continuous case: Cake cutting.