RESEARCH STATEMENT

T. TAM NGUYEN PHAN

I have done work in the general area of Riemannian Geometry and Geometric Topology that I will describe below.

1. Most recently I have been studying noncompact, negatively/nonpositively curved Riemannian manifolds of finite volume, which I will discuss in Section 1. I study the topology of ends of these manifolds and I have constructed new examples of manifolds of this type. Theorem 2 is my best work so far.
2. Before that I studied *rigidity* aspects of spaces obtained by gluing locally symmetric spaces together, which is what my thesis is about. I used the reflection group trick to construct a new class of closed *aspherical* manifolds, called *piecewise locally symmetric manifolds* ([13]), many of which are examples of aspherical manifolds that do not admit a locally CAT(0) metric. I will discuss these manifolds in Section 2. I am generally interested in constructing examples of aspherical manifolds.
3. Locally symmetric manifolds of noncompact type (i.e. those with nonpositive curvature) turn out to have played important roles in my research as they are good examples of many nonpositively curved phenomena. Understanding specifics about these manifolds can be valuable and they are interesting in their own right. One piece of work that I (joint with Grigori Avramidi) have done is to find maximal flat tori in $\mathrm{SO}(n)\backslash \mathrm{SL}(n,\mathbb{R})/\Gamma$, where Γ is a congruence subgroup of $\mathrm{SL}(n,\mathbb{Z})$, that give many nontrivial rational homology cycles. I will discuss this in Section 3.

1. Topology of nonpositively curved ends

Let M^n be a noncompact, complete, finite volume, Riemannian manifold. Gromov ([11]) proved that if the curvature K is negative and bounded, i.e after appropriate scaling of the metric we have $-1 < K < 0$, then M is *tame* in the sense that it has finitely many ends and each end of M is topologically a product $C \times (0,\infty)$. In other words, M is diffeomorphic to the interior of a compact manifold $\overline{M}$ with boundary.

The mechanism for tameness is to show that the injectivity radius function on M does not have any critical point outside a compact set, which can be taken to be the thick part[1] $M_{\geq\epsilon}$ for some *small* $\epsilon > 0$ depending on M. It follows that the thin part $M_{<\epsilon}$ has finitely many components[2] and each component is topologically a product of a closed $(n-1)$-manifold with a ray. The thick-thin decomposition finite volume, hyperbolic manifolds is an example of Gromov's theorem. We call each end $C \times (0,\infty)$ of M a *cusp* and each component of the boundary $\partial\overline{M}$ a *cusp cross section* of M.

[1]For a manifold M and $\epsilon > 0$, the *thick part* $M_{\geq\epsilon}$ of M is the set of points in M with injectivity radius $\geq \epsilon$, and the *thin part* $M_{<\epsilon}$ of M is the set of points in M with injectivity radius $< \epsilon$.

[2]By finiteness of volume, the thick part $M_{\geq\epsilon}$ is compact and therefore has finitely many boundary components.

1.1. **Cusps as topological obstructions to admitting negatively curved metrics.** The topology of cusps gives obstructions to M admitting different types of negatively curved metrics. For example:

a) If M has a negatively curved Riemannian metric with K *pinched*, i.e. $-1 < K < -a^2 < 0$, then each cusp cross section C of M are homeomorphic to compact infra-nilmanifolds ([7], [9]) and thus $\pi_1(C)$ is virtually nilpotent.
b) If $-1 < K < 0$ (but K may tend to 0), then the cusp cross sections must have zero Euler characteristic ([8, Theorem 1.2]) and also zero simplicial volume ([12, p. 17]). These are results in bounded geometry.

   What is more intriguing is that there are manifolds M with $-1 < K < 0$ and $K \to 0$ that do not admit pinched negatively curved metrics. An interesting example is those constructed by Fujiwara ([10]) of manifolds M^n with $-1 \leq K < 0$, in each dimension $n \geq 4$, with cusp cross sections C that are circle bundles over a closed hyperbolic manifold, and therefore $\pi_1(C)$ is not virtually nilpotent.

Remark. The situation is wildly different when the curvature is negative but unbounded (i.e. $K \to -\infty$) because hardly anything can be said. In ([15]), I constructed examples of M with $K < 0$, for each dimension $n > 2$, with cusp cross section a compact hyperbolic manifold, which gives a sharp contrast with the restriction on the simplicial volume and the Euler characteristic of the cusp cross section C when $-1 < K < 0$ as mentioned above. I also constructed examples of finite volume manifolds with unbounded negative curvature that has infinite topological type (i.e. not homotopy equivalent to a finite complex). These are interesting but minor results. A more surprising example, also constructed in [15], is

Theorem 1. *Let Σ_g be a closed surfaces of genus $g \geq 2$. Then the manifold $\mathbb{R} \times \Sigma_g$ has a complete, finite volume, negatively curved Riemannian metric.*

1.2. **Some difference and similarity between $K < 0$ and $K \leq 0$.** Suppose that M has only one end for simplicity in the following discussion.

1. **Difference**: Gromov's theorem is not true when the $-1 < K < 0$ condition is replaced by $-1 < K \leq 0$ by a counterexample due to him. This illustrates how allowing the curvature to be equal to zero can change the situation qualitatively, and we should treat $K < 0$ rather differently from $K \leq 0$.

   Among the locally symmetric spaces of noncompact type, the higher rank [3] manifolds, e.g. those with non-periodic 2-flats, are rather different from the locally symmetric spaces with rank one, e.g. those with negative curvature. The following examples illustrates the difference in the topology of their ends. Let $\widetilde{M}_{<\epsilon}$ be a lift of the thin part $M_{<\epsilon}$ in the universal cover $\widetilde{M}$. It is clear that $M_{<\epsilon}$ is a quotient of $\widetilde{M}_{<\epsilon}$.
   - **Rank-1 examples**: If M is a noncompact, finite volume, hyperbolic manifold, then $\widetilde{M}_{<\epsilon}$ is a horoball, which is contractible.
   - **Higher-rank examples**: If M is a nontrivial product M of noncompact, finite volume, hyperbolic surfaces, then $\widetilde{M}_{<\epsilon}$ a union of overlapping horoballs and, in particular, *not* contractible.

[3]for the right rank.

In fact, Borel and Serre ([6]) proved that $\widetilde{M}_{<\epsilon}$ is homotopy equivalent to the spherical Tits building at infinity, which is a $(k-1)$-dimensional complex, where k is the rational rank of M, that is homotopy equivalent to a wedge of $(k-1)$-spheres. So it looks like the flats in $\widetilde{M}$ that create nontrivial topology of $\widetilde{M}_{<\epsilon}$.

2. **Similarity**: The manifolds in the counterexample mentioned in (1) have arbitrarily small geodesic loops. If M is a noncompact, complete Riemannian manifold with curvature $-1 < K \leq 0$ and finite volume, and if we assume that M does not have arbitrarily small geodesic loops, then M is also *tame* ([4]) by a similar argument as in [11]. Examples of such manifolds are locally symmetric spaces of noncompact type.

   Although it looks like for locally symmetric M *flats* are responsible for nontrivial topology of $\widetilde{M}_{<\epsilon}$, there are negatively curved manifolds M with $\widetilde{M}_{<\epsilon}$ not covered by just one horoballs and not contractible. In ([1]), Abresch and Schroeder constructed examples of noncompact, finite volume manifolds M with $-1 < K < 0$ whose ends are "generalized graph manifolds". In these examples, $\widetilde{M}_{<\epsilon}$ has a similar qualitative structure, i.e. a union of overlapping horoballs, as that of products of surfaces.

The point of the above discussion: Despite the difference, bounded nonpositively curved manifolds (with no small geodesic loops) and bounded negatively manifolds are more similar than they are different. What is qualitatively true for bounded negatively manifolds might be also true for nonpositively curved manifolds with no arbitrarily small geodesic loops. That is, "negatively curved" can be very different from "pinched negatively curved".

1.3. **Topological types of nonpositively curved ends.** Let M be a noncompact, finite volume manifold with $-1 < K \leq 0$. Assume that M has no arbitrarily small geodesic loops. The topology of the thin part $M_{<\epsilon}$ and $\widetilde{M}_{<\epsilon}$ are not well understood. We will call $\widetilde{M}_{<\epsilon}$ the thin part of $\widetilde{M}$. It is the topology of $\widetilde{M}_{<\epsilon}$ that will be discuss in this section.

As mentioned above, when M is locally symmetric, Borel and Serre showed that $\widetilde{M}_{<\epsilon}$ is homotopy equivalent to a $(k-1)$-dimensional complex, where k is the (rational) rank of M. The rank k is at most $n/2$ with $k = n/2$ when M is a product of non-compact surfaces. While this is a very nice fact, one desires more understanding. In recent joint work with Grigori Avramidi([3]), we show that this is no arithmetic coincidence!

Theorem 2 (joint with G. Avramidi, 2016)**.** *Let M be a noncompact, complete, Riemannian manifold with bounded nonpositive sectional curvature $-1 < K \leq 0$ and finite volume. Assume that M has no arbitrarily small geodesic loops. Let $\epsilon > 0$ be small enough so that $M_{<\epsilon}$ is a topologically a product with a ray.*

Let P be a finite polyhedron and let $\varphi\colon P \to \widetilde{M}_{<\epsilon}$ be a continuous map. Then φ can be homotoped within $\widetilde{M}_{<\epsilon}$ to a map $\widehat{\varphi}\colon P \to \widetilde{M}_{<\epsilon}$ such that $\widehat{\varphi}$ factors through a polyhedron Q of dimension less than $\lfloor n/2 \rfloor$. That is, $\widehat{\varphi} = \tau \circ \pi$, for

$$P \xrightarrow{\pi} Q \xrightarrow{\tau} \widetilde{M}_{<\epsilon}.$$

Remark. One cannot do better (i.e. lower) than $\lfloor n/2 \rfloor - 1$. Think about products of non-compact surfaces.

Corollary 3. *The homology of* $\widetilde{M}_{<\epsilon}$ *vanishes in dimension* $\geq \lfloor n/2 \rfloor$, *i.e.*

$$H_{\geq \lfloor n/2 \rfloor}(\widetilde{M}_{<\epsilon}) = 0. \quad (1)$$

Another consequence of (1) is that the geometric dimension[4], denoted $\mathrm{gdim}(\Gamma)$, of Γ is at least $\lceil n/2 \rceil$, where $\lceil n/2 \rceil$ denotes the smallest integer greater or equal to $n/2$.

Corollary 4. *The geometric dimension of* Γ *is* $\geq \lceil n/2 \rceil$,

Questions to ask after (the proof of) Theorem 2.

Question 5. *Is* $\widetilde{M}_{<\epsilon}$ *homotopically equivalent to a complex of dimension less than* $\lfloor n/2 \rfloor$*?*

Question 6. *Does* M *in Theorem 2 admit some stratification similar to the Borel-Serre compactification of locally symmetric spaces?*

When I told people about Theorem 2, I sometimes got the following question, which I think is due to the (false) belief/expectation caused by taking what happens with locally symmetric spaces too far.

Question 7. *Is* $\pi_1(M)$ *a duality group?*

I will resolve this confusion by constructing examples for which $\pi_1(M)$ has nontrivial homology in all dimension $< \lfloor n/2 \rfloor$. I have thought about this question and it is doable.

It is crucial to construct new manifolds. In particular, there is a lack of negatively curved examples so it is important to construct more. This leads us to the next section.

1.4. **The importance of constructing new manifolds.** If we do believe that there is no qualitative difference between bounded negative curvature and bounded nonpositive curvature, the question still remains: are they really the same? That is, can any nonpositively curved cusp be realized as a negatively curved cusp? There is a lack of negatively curved examples so it is important to construct more.

Locally symmetric spaces are good examples for finitely volume, nonpositively curved manifolds with no small geodesic loops. However, there is a big gap between the examples known and the topological restrictions. The Abresch-Schroeder manifolds mentioned above are by far the most interesting examples that are not locally symmetric, but more examples are needed in order to get a better picture of these manifolds.

The question of constructing anything new is very hard in general. But one can start at a more concrete, low-dimensional case, where things might be *approachable* (which is not to be confused with *doable*). When M has dimension 3, then C must be a torus since it is the only oriented surface with zero Euler characteristic. So dimension 4 is to lowest nontrivial dimension to study this question explicitly. This leads us to the next section.

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[4]The *geometric dimension* of Γ is the minimum dimension of an aspherical complex $B\Gamma$ with fundamental group Γ.

1.5. **Classifications of cusps in dimension four.** Given Theorem 2, it follows that when M has dimension 4 the thin part $M_{<\epsilon}$ is aspherical and therefore prime. Due to the Geometrization Theorem, it is now known that all closed 3-manifolds decompose into geometric pieces. If C is a cusp cross section of M, then C has a torus decomposition into pieces that admit one of the following geometries: $\mathbb{E}^3$, Nil, Sol, $\mathbb{H}^3$, $\mathbb{H}^2 \times \mathbb{R}$, and $\widetilde{\mathrm{PSL}(2,\mathbb{R})}$.

Question 8. *Which compact 3-manifolds can be realized as cusp cross section of a noncompact, finite volume, bounded, negatively curved manifolds?*

A closed geometric 3-manifold C with geometry either $\mathbb{E}^3$ or Nil can be realized as a cusp cross section of a finite volume manifold with curvature $-1 < K < 0$ by recent results of Ontaneda ([16]). If C has a torus decomposition with a piece with $\mathbb{H}^3$-geometry, then C cannot be realized as a cusp cross section (this follows from a result in J. Souto's thesis). The case where C is a circle bundle over a surface of genus $g \geq 2$ (in which case C has either $\mathbb{H}^2 \times \mathbb{R}$ or $\widetilde{\mathrm{PSL}(2,\mathbb{R})}$ geometry) has been studied by I. Belegradek. In [5], Belegradek proved that in many cases, such a C can be realized as a cusp cross section of a finite volume manifold with curvature $-1 < K < 0$.

Recent progress. I showed that a closed 3-dimensional manifold C with Sol geometry can occur as cross section of a cusp of a finite volume manifold M with curvature $-1 < K < 0$ ([14]).

Approach. The problem of realizing a cusp cross section of a finite volume manifold with curvature $-1 < K < 0$ can be approached by solving two smaller problems.

1. First, one can try to put a finite volume Riemannian metric with curvature $-1 < K < 0$ on $C \times [0, \infty)$.
2. Then one can try to glue to $C \times [0, \infty)$ a compact negatively curved manifold $\overline{M}$ with boundary C. By recent technology of Riemannian hyperbolization due to Ontaneda ([16]), one can find such $\overline{M}$. The problem is to interpolate between the metric on $C \times [0, \infty)$ found in step 1 and the metric on $\overline{M}$ without changing the sign of the curvature. One hopes that if one warps the metric along the gluing enough, the sign of the curvature will not be altered.

2. Gluing locally symmetric spaces: asphericity and rigidity

We saw above that if X is a locally symmetric manifold, then X has tamed ends. It turns out that the ends of X have a rich enough structure that enables X to admit a compactification $\overline{X}$ that is a manifold with corners by work of Borel and Serre ([6]). The manifold $\overline{X}$ is topologically a manifold with boundary but smoothly a manifold with corners. One can glue two copies of $\overline{X}$ along their boundary to obtain a new closed manifolds but these are not as interesting[5] as gluing many copies of $\overline{X}$ around its corners via the reflection group trick, which I will next explain in general terms. A good example to keep in mind is when X is a product of two punctured tori.

Treat $\overline{X}$ as a room and each codimension one boundary stratum of $\overline{X}$ as a wall that is covered by a mirror. The trick is that mirror reflects light, so when standing in $\overline{X}$ one has the illusion

[5] "Interesting" is subjective.

of being in a world made of many copies of the room $\overline{X}$. One must pick appropriate angles at which the mirrors meet for the illusion to make sense (so that a fool won't get suspicious until he (or she) walks into a mirror). For example, we can decide that when two mirrors meet they meet at a right angle (this corresponds to a right angled Coxeter group). The illusion is a manifold tiled with copies of $\overline{X}$. This manifold is noncompact if it has infinitely many tiles, in which case it is possible to take a quotient that respect the tiling to obtain a closed manifold tiled by finitely many copies of X. We call manifolds obtained in such a way *piecewise locally symmetric manifolds.* These manifolds are interesting in several ways. Firstly, they are *aspherical.*

Theorem 9. *Piecewise locally symmetric manifolds are aspherical despite that fact that many of these do not admit a locally* CAT(0) *metric*[6].

Secondly, these manifolds are rigid. In particular, the embedding of each locally symmetric piece M_i into M are rigid in the following sense ([13, Theorem 3]).

Theorem 10. *Let M be a piecewise locally symmetric manifold of dimension $n > 2$. Suppose that the locally symmetric pieces M_i's of M are irreducible. Let $f\colon M_i \longrightarrow M$ be a π_1-injective map. Then f is homotopic to a diffeomorphism $g\colon M_i \longrightarrow M$ that is onto a piece $M_j \subset M$.*

Thus the decomposition of M into locally symmetric pieces is unique. In other words, up to homotopy there is only one way to tile M by locally symmetric pieces. With the Mostow-Prasad-Margulis rigidity theorem, I also obtained (with some more work) the following rigidity result. Let $\mathrm{Out}(\pi_1(M))$ be the group of outer automorphisms of $\pi_1(M)$. Since M is aspherical by Theorem 9, $\mathrm{Out}(\pi_1(M))$ is the group of self homotopy equivalences of M (up to homotopy).

Corollary 11. *Let M be as in Theorem 10. Then*

a) *Any homotopy equivalence is homotopic to a homeomorphism.*
b) *Moreover, the group of self homotopy equivalences (up to homotopy) of M is realized as a group of homeomorphism. That is, there is lift of* $\mathrm{Out}(\pi_1(M))$ *to* $\mathrm{Homeo}(M)$*, i.e. an injective homomorphism* $\rho\colon \mathrm{Out}(\pi_1(M)) \longrightarrow \mathrm{Homeo}(M)$ *such that*

$$\begin{array}{ccc} & & \mathrm{Homeo}(M) \\ & \overset{\rho}{\nearrow} & \big\downarrow p \\ \mathrm{Out}(\pi_1(M)) & \xleftarrow{\ \eta\ } & \mathrm{Homeo}(M)/\,\mathrm{Homeo}_0(M) \end{array}\ .$$

Corollary 11 is motivated by the following conjecture of Borel. This is a selling point but not exactly the heart of the work.

Conjecture 12 (Borel)**.** *If M is a closed aspherical manifold, and $h\colon N \longrightarrow M$ is a homotopy equivalence, then h is homotopic to a homeomorphism $N \longrightarrow M$.*

We also obtain, in proving Corollary 11, the structure of $\mathrm{Out}(\pi_1(M))$. Any self homotopy equivalence can be obtained the following way. First, we decide which locally symmetric piece goes to which locally symmetric piece. Then we glue them together, and there are some degrees of freedom in doing this. We can "*twist*" as we glue the boundary strata along a loop[7] that

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[6]Some of these manifolds has nilpotent subgroups of the fundamental group that are not virtually abelian.

[7]Think of Dehn twists for surfaces.

corresponds to an element in the center of a certain subgroup of the fundamental group of the boundary. So the group $\mathrm{Out}(\pi_1(M))$ contains a free abelian normal subgroup $\mathcal{T}(M)$ whose elements are *twists*. The quotient is a group $\mathcal{A}(M)$, the elements of which are called *turns*. These can be thought of geometrically as what happens in the first step. So in the case where M has finitely many pieces, $\mathrm{Out}(\pi_1(M))$ is an extension of a finitely generated, torsion free, abelian group $\mathcal{T}(M)$ by a finite group $\mathcal{A}(M)$.

Theorem 13. *Let M be as in Theorem 10. Then $\mathrm{Out}(\pi_1(M))$ is an extension of a free abelian group $\mathcal{T}(M)$ by a group $\mathcal{A}(M)$, i.e. the following sequence is exact.*

$$1 \longrightarrow \mathcal{T}(M) \longrightarrow \mathrm{Out}(\pi_1(M)) \longrightarrow \mathcal{A}(M) \longrightarrow 1.$$

If M has finitely many pieces, then $\mathcal{T}(M)$ is finitely generated and $\mathcal{A}(M)$ is finite.

Remark. There are examples of piecewise locally symmetric spaces with nontrivial twists. In particular, for these manifolds, $\mathrm{Out}(\pi_1(M))$ is infinite[8].

3. Flat cycles in the homology of $\mathrm{SO}(n)\backslash\mathrm{SL}(n,\mathbb{R})/\Gamma$

Let M be a locally symmetric manifold of noncompact type. Totally geodesic submanifolds of M may give information about the topology of M. In particular, they are natural candidates for nontrivial homology cycles. The simplest such submanifolds are maximal periodic tori, i.e. isometrically embedded flat closed tori that are quotients of a maximal flat $\widetilde{T}$ in $\widetilde{M}$. These exist by work of Prasad and Raghunathan. Pettet and Souto showed that these tori cannot be homotoped out to the end ([17]), which leads one to suspect that such tori might be homologically nontrivial. In joint work with Grigori Avramidi ([2]), we show that these tori give an abundance of rationally nontrivial homological cycles.

Theorem 14 (joint with Grigori Avramidi)**.** *Let T be a $(k-1)$-dimensional flat whose image in $M := \mathrm{SO}(n)\backslash\mathrm{SL}(n,\mathbb{R})/\mathrm{SL}(n,\mathbb{Z})$ is compact. Then there is a finite cover M' of M such that the image of T in M' is a nontrivial homology cycle in $H_{k-1}(M',\mathbb{Q})$.*

The mechanism of finding these nontrivial homology cycles is first finding a totally geodesic copy Y of $\mathrm{SO}(n-1)\backslash\mathrm{SL}(n-1,\mathbb{R})$ that is defined over $\mathbb{Q}$ in $\widetilde{M}$ that intersects a flat $\widetilde{T}$ (with compact quotient) transversally in a single point, and then we find a finite index subgroup of $\mathrm{SL}(n,\mathbb{Z})$ such that the images of $\widetilde{T}$ and Y are embedded orientable submanifolds intersecting transversally, with all intersection points having the same sign. The first step can be reduced to showing that the boundaries at infinity of $\widetilde{T}$ and Y are linked, which is not a delicate condition. In fact, we can find many such nontrivial homology cycles as one goes up in covers.

Theorem 15. *Let Γ be a finite index torsion free subgroup of $\mathrm{SL}_n\mathbb{Z}$. Let p be a prime and let $\Gamma(p^k) := \Gamma \cap \ker(\mathrm{SL}_n\mathbb{Z} \to \mathrm{SL}_n(\mathbb{Z}/p^k))$ the p^k congruence subgroup. Then for each $N > 0$, there is k_0 such that for $k \geq k_0$, the dimension of the subspace of $H_{n-1}(\Gamma(p^k)\setminus H;\mathbb{Q})$ spanned by flat cycles is $\geq N$.*

[8]Contrast this with irreducible locally symmetric spaces of dimension ≥ 3, which have finite outer automorphism groups of the fundamental group.

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