Homework #2

(February 22, 2016)

---

1. Let F be a c.d.f. satisfying $0 < F(0-) \leq F(0) < 1$, and $X_1, X_2, \ldots, X_n$ be iid samples from F. Prove that
$$\lim_{n\to\infty} P\left(\max_{1\leq i\leq n} X_i > 0 \text{ and } \min_{1\leq i\leq n} X_i < 0\right) = 1.$$

2. Let $X_1, \ldots, X_n$ be iid random samples from a common cdf F. Let $F_n(x)$ be the empirical cdf based on X_i's. Define $F_n^\star(x) = F_n(x) - c\bar{X}$, where c is constant and $\bar{X}$ is the sample mean. Find conditions such that $F_n^\star(x)$ is an unbiased estimator of $F(x)$. If $F_n^\star(x)$ is an unbiased estimator of $F(x)$, determine c such that $F_n^\star(x)$ is asymptotically efficient, i.e., it achieves minimum asymptotic variance.

3. Let $X_1, \ldots, X_n$ be iid random samples from a common cdf F. Let $F_n(x)$ be the empirical cdf based on X_i's. For a given $x \in \mathbb{R}$, find the limiting distribution of $F_n(x)^2$, $\sqrt{F_n(x)}$ and $\exp(F_n(x))$. Hint: use delta-method.

4. Let x and y be two distinct real numbers. Find $Cov(F_n(x), F_n(y))$, where F_n is empirical cdf.

5. Prove that the empirical cdf F_n based on distinct samples X_i's satisfies
$$F_n(X_i) - F_n(X_i-) = \frac{1}{n},\ \forall n = 1, \ldots, n.$$