Homework #1

(February 4, 2016)

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1. Prove Corollary 2.2
2. Prove Corollary 2.3.
3. Suppose kernel K satisfies condition (K1). Prove that K^2 is integrable, and $\lim_{|y|\to\infty} yK^2(y) = 0$.
4. Let g and h be integrable functions on $\mathbb{R}$, and h satisfy $\lim_{|y|\to\infty} yh(y) = 0$. Prove that

   (a) If g is continuous at x, then

   $$\lim_{b\to 0} \int_{\mathbb{R}} |g(x-tb) - g(x)| \cdot |h(t)| dt = 0.$$

   (b) If g is uniformly continuous, then

   $$\lim_{b\to 0} \sup_{x\in\mathbb{R}} \int_{\mathbb{R}} |g(x-tb) - g(x)| \cdot |h(t)| dt = 0.$$

   Hint: By uniform continuity we mean that

   $$\lim_{\delta\to 0} \sup_{x,y\in\mathbb{R}:|x-y|\le\delta} |f(x) - f(y)| = 0.$$

5. Suppose f is a uniformly continuous probability density, and kernel K satisfies both (K1) and (K2). Furthermore, as $n \to \infty$, $b \to 0$ and $n^{\xi} b^2 \to \infty$, where $\xi \in (0,1)$ is a fixed constant. Prove that

   $$\sup_{x\in\mathbb{R}} |\widehat{f}(x) - f(x)| \to 0, a.s.,$$

   where $\widehat{f}$ is a kernel density estimator of f based on K.