Math 447 Practice Final 2

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Explain your answers and reasoning.

Name: ______________________________

ID number: ______________________________

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	15	10	10	10	10	10	10	10	10	95
Score:										

Instructions:

- Read problems very carefully. If you have any questions raise your hand.
- The correct final answer alone is not sufficient for full credit - try to explain your answers as much as you can (except for Problem 1). This way you are minimizing the chance of losing points for not explaining details, and maximizing the chance of getting partial credit if you fail to solve the problem completely. However, try to save enough time to seriously attempt to solve all the problems.
- You final answers need to be simplified only if this is required in the statement of the problem. Otherwise, you can leave them in any form you wish.
- If you need more paper please raise your hand (you can write on the back pages).
- There is no need to simplify numerical quantities like $4 \cdot 0.3 \cdot \frac{0.3+\frac{3}{4}}{\frac{7}{5}+\frac{23}{547}}$ or to reduce fractions like $\frac{3}{6}$ to lowest terms. There is also no need to simplify factorials such as 5! or binomial coefficients such as $\binom{7}{3}$.

1. (a) (5 points) Random variable X has the normal distribution with the mean 2 and variance 9. Using the attached table, estimate the probability $\mathbf{P}(X \geq -1)$.

   (b) (5 points) Let X and Y be independent random variables with such that $\mathbf{E}[X] = \mathbf{E}[Y] = 1$ and $\text{var}[X] = \text{var}[Y] = 2$. Let $Z = X - Y$. Compute $\mathbb{E}[ZX]$.

   (c) (5 points) If events A and B satisfy $\mathbf{P}(A) = 0.5$ and $\mathbf{P}(A \cap B) = 0.2$, find the conditional probability $\mathbf{P}(B|A)$.

2. (10 points) You have three boxes. The first one contains 1 white and 8 black balls, the second one contains 5 white and 4 black balls, and the last one contains 2 white and 1 black ball. You choose one of these three boxes uniformly at random, and then pick a ball from this box also uniformly at random. What is the probability you pick a white ball?

3. (10 points) You randomly rearrange the numbers $1, 2, \ldots, 10$ so that all possible arrangements are equally likely.

   (a) What is the probability your rearrangement is $10, 9, 8, 7, 6, 5, 4, 3, 2, 1$?

   (b) What is the probability 6 appears in your rearrangement before 3? Do not leave your answer as a sum.

4. (10 points) Let (X, Y) be distributed on the set $[1,2] \times [1,2]$ with pdf

$$f_{X,Y}(x, y) = \frac{c}{xy}.$$

(a) What is c?

(b) Compute $\mathbf{E}[X + Y]$.

(c) Are X,Y independent?

5. (10 points) Let T be an exponentially distributed random variable with parameter 1, and conditioned on $T = t$ let X be a uniformly distributed random variable on $[0, 1/t]$. Compute the pdf of X.

6. (10 points) Let X be an exponential random variable with parameter λ and let Y be an exponential random variable with parameter μ. Assume X and Y are independent. Compute the probability that $X < Y$.

7. (10 points) Let X and Y be independent random variables such that X is a uniform random variable on the interval $[0, 1]$ and Y has pdf

$$f_Y(y) = \begin{cases} 0, & \text{if } y < 1, \\ y^{-2}, & \text{if } y \geq 1. \end{cases}$$

Compute the pdf of $Z = X + Y$.

8. (10 points) Let X be a uniform random variable on the interval $[-1, 3]$. Let $Y = X^2$. Compute the pdf of Y.

9. (10 points) Toss a fair coin 4 times. Then roll a fair six-sided die for each time coin was a head. Let X be the sum of the dice rolls.

   (a) Compute $\mathbb{E}[X]$.

   (b) Compute $\mathrm{Var}[X]$.

   Recall the variance of uniformly distributed discrete random variable on the interval $[a, b]$ has variance $\frac{(b-a)(b-a+2)}{12}$.