Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Explain your answers and reasoning.

Name: ____________________

ID number: ____________________

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	0	10	10	10	10	10	10	90
Score:											

Instructions:

- Read problems very carefully. If you have any questions raise your hand.
- The correct final answer alone is not sufficient for full credit - try to explain your answers as much as you can (except for Problem 1). This way you are minimizing the chance of losing points for not explaining details, and maximizing the chance of getting partial credit if you fail to solve the problem completely. However, try to save enough time to seriously attempt to solve all the problems.
- You final answers need to be simplified only if this is required in the statement of the problem. Otherwise, you can leave them in any form you wish.
- If you need more paper please raise your hand (you can write on the back pages).
- There is no need to simplify numerical quantities like $4 \cdot 0.3 \cdot \frac{0.3+\frac{3}{4}}{\frac{7}{5}+\frac{23}{547}}$ or to reduce fractions like $\frac{3}{6}$ to lowest terms. There is also no need to simplify factorials such as 5! or binomial coefficients such as $\binom{7}{3}$.

1. (a) (2 points) Random variable X has the normal distribution with the mean -1 and variance 16. Using the attached table, estimate the probability $\mathbf{P}(X \geq -2)$.

   (b) (2 points) If X has binomial distribution with parameters n and p, what is the expectation and variance of X. You can just state the answer, no need to prove it.

   (c) (2 points) If random variables Y and Z have binomial distributions (with some parameters), can $Y + Z$ have geometric distribution? Justify your answer.

   (d) (2 points) Let X and Y be independent random variables with zero means, that is $\mathbf{E}[X] = \mathbf{E}[Y] = 0$. Show that

$$\text{var}(X + Y + XY) = \text{var}(X) + \text{var}(Y) + \text{var}(XY).$$

   (e) (2 points) In a sample space S we have n events $A_1, A_2, \ldots, A_n$, each having probability equal to p. If $n \geq 3$ and $p > 2/n$, show that there is an sample point (simple event) $E \in S$ which is contained in at least three of the above events.

2. A box contains $2n$ red and $2n$ blue toys. We select uniformly at random $2n$ toys from the box.

   (a) (5 points) Compute the probability that we selected equally many red and blue toys.

   (b) (5 points) Given that we selected more red than blue toys, compute the conditional probability that no blue toys were selected.

3. We have n fair coins at our disposal. We perform the experiment in which we toss these n coins independently. We discard the coins which came up tails, and keep those which came up heads. If we have no more coins (that is, if all coins came up tails) we end the experiment. Otherwise, we repeat the experiment with the coins which are left, so we toss the coins which are left and discard those which came up tails. We repeat this experiment until we have no more coins left. Let X denote the number of times we had to perform the experiment.
   (a) (5 points) Given that $X = 2$, compute the conditional probability that in the first experiment exactly one coin landed heads.
   (b) (5 points) Find the PMF of X.

4. Let the random variables X and Y be jointly continuous with the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} ke^{-(ax+by)}, & \text{if } x > 0, y > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $a, b > 0$ and k is a constant.

1. Find k.
2. Are X and Y independent?

5. In a certain game a computer prints a random positive integer number N whose distribution is Poisson with parameter λ. Then another computer selects a real number X uniformly in the interval $[0, N+1]$.

   (a) (5 points) Compute the PDF of X at all values between 0 and 1, that is compute $f_X(x)$ for all $0 < x < 1$.

   (b) (5 points) Compute the expectation of X.

6. Random variable X has PDF given by the formula

$$f_X(x) = \begin{cases} x^{-2}, & \text{if } x \geq 1, \\ 0, & \text{if } x < 1. \end{cases}$$

Random variable Y is uniform in the interval $[0, 1]$. Assume that random variables X and Y are independent.

(a) (7 points) For any real number $t > 0$ compute $\mathbf{P}(XY \leq t)$.

(b) (3 points) Find the PDF of $Z = XY$.

7. Select a point (X, Y) uniformly from the region which lies above the x-axis, to the right of the y-axis, to the left of the line $x = 1$ and below the graph of $y = 2 - x^2$.
   (a) (5 points) Compute the marginal PDF of X and the marginal PDF of Y.
   (b) (5 points) Compute the $\mathbf{E}[XY]$. You don't need to simplify the final numerical expression you get.

8. One Tuesday midterms are held in Kerchoff and Boelter halls, between 10AM and 11AM and between 1PM and 2PM (each midterm lasts 60 minutes). Three protesters are going to pull fire alarms on campus that Tuesday, protesters A and B will pull fire alarms in Kerchoff Hall, and protester C will pull fire alarm in Boelter Hall. Each of these three protesters will pull fire alarm at a uniform time between 8AM and 8PM, independently of other protesters. If the alarm is activated during midterm time, all the students who are writing the midterm at the time will have to leave.

   (a) (4 points) Compute the probability that no midterms in Kerchoff Hall are interrupted.

   (b) (6 points) The Monday before the midterms, an administrator in the Boelter Hall found out about protesters plan and decided to perform a drastic (and probably illegal) measure. He ordered all the fire alarms in Boelter Hall to be disabled, so that when protester C pulls the fire alarm, it doesn't go off. However, the crew which will disable the fire alarm will arrive on Tuesday at a random time which is uniformly distributed between 6AM and 6PM. This time is independent of the time when the protester C comes to Boelter Hall. Compute the probability that no midterms in Boelter Hall are interrupted.

9. (10 points) Let X be an exponential random variable with parameter 1. Let Y be a uniform random variable on the interval $[1, 3]$. If X and Y are independent. Compute the pdf of $Z = X + Y$.

10. (10 points) You are running an illegal gambling parlor. In one of your games of dice, you collect 1 dollar with probability $p = 3/4$ and lose 1 dollar with probability $1/4$. You play N games of dice, where N is a geometric random variable with mean 10, until the cops end your game. Let T be the amount of money you win after N games. What are $\mathbf{E}[T]$, $\text{var}[T]$?

    (A geometric random variable with mean 10 (or parameter 1/10) has variance 90.)