

## Homework 6

Do the problems on webwork and turn the following problems in class on Mar. 15st.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

**Problem 1.** Let  $X$  be a random variable with moment generating function (MGF):

$$m_X(t) = \frac{e^{2t}(-1 + e^{5t})^2}{25(-1 + e^t)^2}$$

(a) Verify that  $m_X(0) = 1$ , a fact that is true for all MGF.

(b) Compute  $\mathbb{E}[X]$ .

(c) Compute  $\text{Var}[X]$ .

(NOTE 1: I strongly recommend using a computer to help with the calculations.)

(NOTE 2: The function  $m_X(t)$  as given technically doesn't exist at 0, but it is a continuous function for  $t$  near zero, so we define  $m_X(0) = \lim_{t \rightarrow 0} m_X(t)$ .)

**Problem 2.** Which of the following functions can be a probability density function for some continuous random variable. Explain your reasoning.

1)

$$f(t) = \begin{cases} \pi \sin(\pi t), & \text{for } 0 \leq t \leq 3/2 \\ 0, & \text{otherwise.} \end{cases}$$

2)

$$g(t) = \begin{cases} 2e^{-t/2}, & \text{for } t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

3)

$$h(t) = \begin{cases} \frac{1}{e-1} e^{\cos(t)} \sin(t), & \text{for } 0 \leq t \leq \pi \\ 0 & \text{otherwise.} \end{cases}$$

4)

$$j(t) = \begin{cases} 0, & \text{for } t < 0 \\ 1, & \text{for } 0 \leq t \leq 1/2 \\ 0, & \text{for } 1/2 < t < 1 \\ 1, & \text{for } 1 \leq t \leq 3/2 \\ 0, & \text{for } 3/2 < t. \end{cases}$$

5)

$$k(t) = \begin{cases} t^2, & \text{for } -(3/2)^{1/3} \leq t \leq (3/2)^{1/3} \\ 0, & \text{otherwise.} \end{cases}$$