

Homework 5

Do the problems on **Webwork** and turn the following problems in class on Feb. 23th.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

Problem 1. In this problem we will assume there are N , a large number of people in a room, and use the Poisson distribution to estimate the probability at least k of them share a birthday. We assume everyone's birthday is uniformly distributed over the 365 days in a year, independently.

- (a) How many sets of exactly k people are there?
- (b) What is the probability all k people in one of these sets share the same birthday?
- (c) Let X be a Poisson random variable, that approximates the number of sets of k people that all have the same birthday. What parameter λ should we use in our Poisson approximation?
- (d) If $N = 100$ and $k = 3$. What is $\mathbb{P}(\{X = 0\})$? What does this number represent in our approximation?

Problem 2. An insect lays a random number of eggs. The number of eggs has a Poisson distribution with parameter 100. Each egg develops into an offspring with probability $1/2$.

Let Y be the number of offspring.

- (a) Show that Y is also a Poisson random variable. What is its parameter?
- (b) What is $\mathbb{E}[2Y^2 - Y + 1]$?