

Homework 10

Do the problems on **Webwork** and turn the following problems in class **on Wednesday April 11**.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

Problem 1. A group of n people arrives at a circular table with $3n$ chairs. People are seated at n of these chairs chosen uniformly at random. A person will be bored if there is nobody sitting at the chair to the left.

Let X_i for $1 \leq i \leq n$ equal 1 if the i^{th} person is bored and 0 otherwise.

Let $N = \sum_{i=1}^n X_i$ be the number of bored people.

- Compute $\mathbb{E}[X_i]$. Hint: count number of way people can be arranged so that person 1 is bored and count the total number ways the people can be arranged.
- Compute $\text{Var}[X_i]$. Hint: what is kind of random variable is X_i ?
- Compute $\mathbb{E}[N]$.
- Compute $\mathbb{E}[X_i X_j]$ for $1 \leq i \neq j \leq n$. Hint: once again count the number of arrangements where $X_i X_j = 1$ or use conditioning.
- Compute $\text{Var}[N]$.

Problem 2. Suppose N , the number of job requests a company gets in a given week, has geometric distribution with mean 10.

Suppose Y_i , the profit they make on the i^{th} job, has gamma distribution with $\alpha = 4$ and $\beta = 1$, and the Y_i 's are independent.

Let $T = \sum_{i=1}^N Y_i$ be the total profit.

- What is $\mathbb{E}[T|N = n]$?
- What is $\mathbb{E}[T]$?
- What is $\text{Var}[T]$?