

## Homework 10

Do the problems on **Webwork** and upload the following problems to Gradescope before 8 am on April 3rd.

**When you upload your assignment, mark the page on which your solution to each problem starts, or upload each problem individually.**

**Homework should be written neatly and clearly explained. Include your name and id number in the top right corner of your homework.**

**Problem 1.** Which of the following pairs of random variables are independent?

(a) Let  $X_1$  and  $X_2$  have joint pdf

$$f_{X_1, X_2}(x_1, x_2) = \frac{3}{4}x_1^2(1 - x_2) \text{ for } 0 \leq x_1 \leq 2, 0 \leq x_2 \leq 1$$

and 0 otherwise.

(b) Let  $Y_1$  and  $Y_2$  have joint pdf

$$f_{Y_1, Y_2}(y_1, y_2) = 3y_1y_2$$

on the region bounded by the lines  $y_2 = 0$ ,  $y_1 = y_2$  and  $y_1 + y_2 = 2$ .

(c) Let  $Z_1$  and  $Z_2$  have joint pdf

$$f_{Z_1, Z_2}(z_1, z_2) = \frac{1}{6\pi} e^{-\left(\frac{z_1^2}{4} + \frac{z_2^2}{9}\right)}$$

for all  $z_1, z_2$ .

**Problem 2.** A person has a highly contagious disease. The number of people they meet each day is a Poisson random variable with mean 5. They infect each person they meet with probability  $1/3$ , independently.

Let  $Y$  be the number of people they meet and  $X$  be the number they infect.

(a) Conditioned on the event that  $\{Y = n\}$ , what is the pmf of  $X$ ? (Your answer should involve  $n$ ).

(b) What is the joint distribution of  $X$  and  $Y$ ? (Don't forget the bounds.)

(c) Show that  $X$  is a Poisson random variable. What is  $\mathbb{E}[X]$ ?