

Homework 5 - Due Friday, Oct. 5

Do Problems 8.89, 8.95, 8.100, 8.103, 9.2, 9.3, 9.7, 9.15, 9.18, 9.21 and the problem below.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

**Problem 1.** An experimental trial produces random variables  $X$  and  $Y$  with covariance  $\sigma_{XY} = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$ . To estimate  $\sigma_{XY}$ , we perform  $N$  independent trials and form the estimator

$$\widehat{R}_N = \frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})$$

where  $X_1, \dots, X_N$  and  $Y_1, \dots, Y_N$  are random samples from a population with the same distribution as  $X$  and  $Y$ , respectively. As usual  $\bar{X} = \frac{1}{N} \sum_{i=1}^N X_i$  and  $\bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_i$ .

Show that if  $\text{Var}[X_i Y_i]$  is finite, then  $\widehat{R}_N$  is an unbiased, consistent estimator of  $\sigma_{XY}$ .

Hint: Why can you assume w.l.o.g.  $\mathbb{E}[X] = \mathbb{E}[Y] = 0$ ?