Homework 8

Do the problems on webwork and turn the following problems in class on April 5th.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

Problem 1. We have a fair coin. In the first stage of the experiment we keep tossing it until we get a heads, and we remember how many times we had to toss it (say k times). In the 2nd stage of the experiment we will toss the same coin the same number of times (k times).

Let X be the number of times the coin was flipped in the first stage and Y be the number of heads in the second stage.

(a) What is the probability mass function of X?

(b) What is the conditional probability mass function of Y given X?

(c) Use part a) and b) to compute the joint probability mass function of X and Y.

(d) What is the probability that in the 2nd stage we get exactly 1 head?

Problem 2. (a) Let X_1 and X_2 have joint probability density function:

$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} 4x_1x_2, & \text{if } 0 \le x_1 \le 1, 0 \le x_2 \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Are X_1, X_2 independent? What is the marginal pdf of X_1 and X_2?

(b) Let Y_1 and Y_2 have joint probability density function:

$$f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} 6(1 - y_1), & \text{if } 0 \le y_2 \le y_1 \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Are Y_1, Y_2 independent? What is the marginal pdf of Y_1 and Y_2?