Homework 10

Do the problems on webwork and turn the following problems in class on WEDNESDAY, April 24th.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

Problem 1. You are operating a train. Ticket for this train costs $10. When the train is late you give a refund under the following rules:

If the train is less than 10 minutes you refund $5, if the train is more than 10 minutes late you refund $12.

If a train is late and the amount of time that it is late has exponential distribution with mean 1, what is the expectation of your profit (income minus refund) per customer?

(Hint: Let L be the time the train is late and P be your profit. Use the conditional expectation of P given L to compute the expectation of P.)

Problem 2. There are N people, labeled $1, 2, \ldots, N$, each with a snow ball, each person chooses a different person uniformly at random and throws a snowball at them, independently. Let X be the the number of people who get hit by at least one snowball. Let X_i be 1 on the event that the ith person gets hit by a snowball and 0 otherwise. Note that the X_i's are not independent, but they do have the same distribution (pmf).

(a) Compute $\mathbb{E}[X_i]$.

(b) Compute $\mathrm{Var}[X_i]$.

(c) Compute $\mathbb{E}[X_i X_j]$.

(d) Compute $\mathbb{E}[X]$.

(e) Compute $\mathrm{Var}[X]$.

I've noticed this problem is actually easier to do if you instead consider Y_i which is 1 on the event that the ith person does NOT get hit by a snowball and 0 otherwise, and then let Y be the number of people who are not hit. That is $Y_i = 1 - X_i$ and $Y = N - X$. Since $\mathbb{E}[Y] = N - \mathbb{E}[X]$ and $\mathrm{Var}[Y] = \mathrm{Var}[X]$, so you can solve the problem using the $X's$ or $Y's$.

You can turn in your answers for the $X's$ and $Y's$.