

EXAMPLE 5.1.5

A manufacturer has found that marginal cost is $3q^2 - 60q + 400$ dollars per unit when q units have been produced. The total cost of producing the first 2 units is \$900. What is the total cost of producing the first 5 units?

Solution

Recall that the marginal cost is the derivative of the total cost function $C(q)$. Thus,

$$\frac{dC}{dq} = 3q^2 - 60q + 400$$

and so $C(q)$ must be the antiderivative

$$C(q) = \int \frac{dC}{dq} dq = \int (3q^2 - 60q + 400) dq = q^3 - 30q^2 + 400q + K$$

for some constant K . (The letter K was used for the constant to avoid confusion with the cost function C .)

The value of K is determined by the fact that $C(2) = 900$. In particular,

$$900 = (2)^3 - 30(2)^2 + 400(2) + K \quad \text{or} \quad K = 212$$

Hence,

$$C(q) = q^3 - 30q^2 + 400q + 212$$

and the cost of producing the first 5 units is

$$C(5) = (5)^3 - 30(5)^2 + 400(5) + 212 = \$1,587$$

EXAMPLE 5.1.7

A retailer receives a shipment of 10,000 kilograms of rice that will be used up over a 5-month period at the constant rate of 2,000 kilograms per month. If storage costs are 1 cent per kilogram per month, how much will the retailer pay in storage costs over the next 5 months?

Solution

Let $S(t)$ denote the total storage cost (in dollars) over t months. Since the rice is used up at a constant rate of 2,000 kilograms per month, the number of kilograms of rice in storage after t months is $10,000 - 2,000t$. Therefore, since storage costs are 1 cent per kilogram per month, the rate of change of the storage cost with respect to time is

$$\frac{dS}{dt} = \left(\begin{array}{c} \text{monthly cost} \\ \text{per kilogram} \end{array} \right) \left(\begin{array}{c} \text{number of} \\ \text{kilograms} \end{array} \right) = 0.01(10,000 - 2,000t)$$

It follows that $S(t)$ is an antiderivative of

$$0.01(10,000 - 2,000t) = 100 - 20t$$

That is,

$$\begin{aligned} S(t) &= \int \frac{dS}{dt} dt = \int (100 - 20t) dt \\ &= 100t - 10t^2 + C \end{aligned}$$

for some constant C . To determine C , use the fact that at the time the shipment arrives (when $t = 0$) there is no cost, so that

$$0 = 100(0) - 10(0)^2 + C \quad \text{or} \quad C = 0$$

Hence,

$$S(t) = 100t - 10t^2$$

and the total storage cost over the next 5 months will be

$$S(5) = 100(5) - 10(5)^2 = \$250$$

EXAMPLE 5.1.6

The population $P(t)$ of a bacterial colony t hours after observation begins is found to be changing at the rate

$$\frac{dP}{dt} = 200e^{0.1t} + 150e^{-0.03t}$$

If the population was 200,000 bacteria when observations began, what will the population be 12 hours later?

Solution

The population $P(t)$ is found by antidifferentiating $\frac{dP}{dt}$ as follows:

$$\begin{aligned} P(t) &= \int \frac{dP}{dt} dt = \int (200e^{0.1t} + 150e^{-0.03t}) dt \\ &= \frac{200e^{0.1t}}{0.1} + \frac{150e^{-0.03t}}{-0.03} + C \quad \text{exponential and sum rules} \\ &= 2,000e^{0.1t} - 5,000e^{-0.03t} + C \end{aligned}$$

Since the population is 200,000 when $t = 0$, we have

$$\begin{aligned} P(0) &= 200,000 = 2,000e^0 - 5,000e^0 + C \\ &= -3,000 + C \end{aligned}$$

so $C = 203,000$ and

$$P(t) = 2,000e^{0.1t} - 5,000e^{-0.03t} + 203,000$$

Thus, after 12 hours, the population is

$$\begin{aligned} P(12) &= 2,000e^{0.1(12)} - 5,000e^{-0.03(12)} + 203,000 \\ &\approx 206,152 \end{aligned}$$

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- 43. MARGINAL COST** A manufacturer estimates that the marginal cost of producing q units of a certain commodity is $C'(q) = 3q^2 - 24q + 48$ dollars per unit. If the cost of producing 10 units is \$5,000, what is the cost of producing 30 units?

44. **MARGINAL REVENUE** The marginal revenue derived from producing q units of a certain commodity is $R'(q) = 4q - 1.2q^2$ dollars per unit. If the revenue derived from producing 20 units is \$30,000, how much revenue should be expected from producing 40 units?

45. **MARGINAL PROFIT** A manufacturer estimates marginal revenue to be $R'(q) = 100q^{-1/2}$ dollars per unit when the level of production is q units. The corresponding marginal cost has been found to be $0.4q$ dollars per unit. Suppose the manufacturer's profit is \$520 when the level of production is 16 units. What is the manufacturer's profit when the level of production is 25 units?

46. **SALES** The monthly sales at an import store are currently \$10,000 but are expected to be declining at the rate of

$$S'(t) = -10t^{2/5} \text{ dollars per month}$$

t months from now. The store is profitable as long as the sales level is above \$8,000 per month.

- Find a formula for the expected sales in t months.
- What sales figure should be expected 2 years from now?



- For how many months will the store remain profitable?

47. **ADVERTISING** After initiating an advertising campaign in an urban area, a satellite dish provider estimates that the number of new subscribers will grow at a rate given by

$$N'(t) = 154t^{2/3} + 37 \text{ subscribers per month}$$

where t is the number of months after the advertising begins. How many new subscribers should be expected 8 months from now?