Sec 3.1 - "u substitution" homework

1a) $\int (3x+1)^5 dx$ let $u = 3x+1$ → so $u' = 3$

, so $\frac{du}{dx} = 3 \rightarrow \boxed{dx = \frac{du}{3}}$

Substitute u terms for x terms: $\int u^5 \frac{du}{3} = \frac{1}{3}\int u^5 du$

Take the antiderivative:

$$\frac{1}{3}\int u^5 du = \frac{1}{3}\frac{u^6}{6} + C = \frac{1}{18}u^6 + C$$

Substitute x-term back into answer:

$$\frac{1}{18}(3x+1)^6 + C = \frac{(3x+1)^6}{18} + C$$

b) $\int (-t+1)^3 dt$ let $u = -t+1$

so $\frac{du}{dt} = -1 \rightarrow \boxed{du = -dt}$

$\boxed{dt = -du}$

Substitute u terms; take antiderivative

$$\int -u^3 du = -\int u^3 du = -\frac{u^4}{4} + C$$

Substitute x-term back:

$$-\frac{(-t+1)^4}{4} + C \quad \text{or} \quad -\frac{(1-t)^4}{4} + C$$

1 e) $\int \sqrt{4x-1}\, dx = \int (4x-1)^{1/2}\, dx$

Let $u = 4x-1$, $n = 1/2$

$\frac{du}{dx} = 4 \rightarrow \boxed{dx = \frac{du}{4}}$ cross (x)

Hence $\int u^{1/2} \frac{du}{4} = \frac{1}{4}\int u^{1/2}\, du = \frac{1}{4}\left(\frac{u^{3/2}}{3/2}\right) + C$

$= \frac{u^{3/2}}{6} + C = \frac{(4x-1)^{3/2}}{6} + C$

d) $\int (-4x^3 + 2x - 1)(\underbrace{x^4 - x^2 + x}_{u})^4\, dx$

$-(4x^3 - 2x + 1)$

Let $u = x^4 - x^2 + x$, $n = 4$

$\frac{du}{dx} = 4x^3 - 2x + 1 \rightarrow du = 4x^3 - 2x + 1\, dx$

Do we have a $\int u^n\, du$? It's off by a negative coefficient of 1.

~~Just cancel, but we need to see the $u^n du$ so we write it~~

$-\int u^4\, du$

However, the original $\int(-4x^3+2x-1)\cdots$

can be factored by -1:

$$-\int(4x^3-2x+1)\,\underline{(x^4-x^2+x)^4\,dx}$$

which is a match when u is sub'd:

$$-\int u^4\,du = -\frac{u^5}{5}+C$$

$$= -\frac{(x^4-x^2+x)^5}{5}+C$$

1; $\int x^1(x^1-2)^5\,dx = \int x\cdot(x-2)^5\,dx$

Notice that substituting $\boxed{u = x-2}$ won't give

$\int u^n\,du$ form.

For this type, $\int x(x\pm c)^n\,dx$

try substituting $u\pm C = x$

then $x\pm C = u$ (roughly)

$\int x(x-2)^5\,dx$ $\quad \boxed{x = u+2}$ ② $\leftarrow$ ① $\boxed{x-2=u}$ let

& $dx = du$

It's a double u substitution, but you want the term in ( ) to be u.

$$\int\underset{x}{(u+2)}\,\underset{x-2}{(u)^5}\,du = \int(u^6+2u^5)\,du$$

$$= \frac{u^7}{7} + \frac{2u^6}{6} + C = \frac{u^7}{7} + \frac{u^6}{3} + C$$

Resubstitute: $\frac{(x-2)^7}{7} + \frac{(x-2)^6}{3} + C$

j) $\int \frac{e^{2x}}{e^{2x}+5}\, dx$ Notice $\cancel{\frac{d}{dx}}(e^x) = e^x\, dx$

so this is roughly a

$\int \frac{du}{u}$ form

Let $u = e^{2x} + 5$

then $du = 2e^{2x}\, dx$

or $\frac{du}{2} = e^{2x}\, dx$

Then $\int \frac{e^{2x}}{e^{2x}+5}\, dx = \int \frac{du}{2u}$.

$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|e^{2x}+5| + C$

k) $\int \frac{\ln x}{x}\, dx$ let $u = \ln x$; $\frac{du}{\cancel{dx}} = \frac{1}{x}\, dx$

$\int u\, du = \frac{u^2}{2} + C = \frac{(\ln x)^2}{2} + C$

1i done again

1i) $\int x(x-2)^5 \, dx$

let $u = x-2$

$\frac{du}{dx} = 1 \rightarrow du = 1dx$

$= \int \cancel{x} \, u^5 \, du$

??

But, since

$\boxed{u = x-2}$

then $x = u+2$

- substitute ~~this~~ also

$x = u+2$

(two u-subs!)

$= \int (u+2)\, u^5 \, du$

$= \int (u^6 + 2u^5) \, du$

$= \frac{u^7}{7} + \frac{2u^6}{63} + C$

$= \frac{(x-2)^7}{7} + \frac{(x-2)^6}{3} + C$

Like #1:

$$\int 4x(x+1)^3\,dx$$

$$= 4\int x(x+1)^3\,dx$$

let $\boxed{u = x+1} \longrightarrow x = u-1$

$$\frac{du}{dx} = 1 \rightarrow du = dx$$

So far: $4\int x\,u^3\,du$

$$= 4\int (u-1)\,u^3\,du$$

$$= 4\int (u^4 - u^3)\,du$$

$$= 4\left(\frac{u^5}{5} - \frac{u^4}{4}\right) + C$$

$$= 4\left(\frac{(x+1)^5}{5} - \frac{(x+1)^4}{4}\right) + C$$

#1l) Redone from class

$$\int \frac{4x}{\sqrt{x^2+9}}\,dx = 4\int x\,(x^2+9)^{-1/2}\,dx$$

let $\boxed{u = x^2+9}$ ← u-subs $(n = -1/2)$

then $\frac{du}{dx} = 2x$, or $\boxed{\frac{du}{2} = x\,dx}$

$$4\int \underbrace{x\,\overbrace{(x^2+9)^{1/2}}^{u^{1/2}}\,dx}_{du/2} = 4\int \frac{u^{-1/2}}{2}\,du$$

$$= \frac{4}{2}\int u^{-1/2}\,du = 2\,\frac{u^{1/2}}{1/2} + C$$

$$= 4u^{1/2} + C = 4(x^2+9)^{1/2} + C$$

#3) $P'(x) = x e^{-x^2}$

$P(x) = 8000$ when $x = 4$

$$P(x) = \int x e^{-x^2}\,dx = -\frac{1}{2}\int \underbrace{-2x\,e^{\overbrace{-x^2}^{u}}\,dx}_{du}$$

$$P(x) = -\frac{1}{2} e^{-x^2} + C$$

$$P(4) = -\frac{1}{2} e^{-(4^2)} + C = 8000$$

$$C = 8000 + \frac{e^{-16}}{2}$$

$$P(x) = -\frac{1}{2} e^{-x^2} + \underbrace{8000}_{M} + \frac{1}{2e^{16}}$$

calculated by .008 million

$$8K \times \frac{1 \text{ million}}{10^3 K} = .008 \text{ million}$$