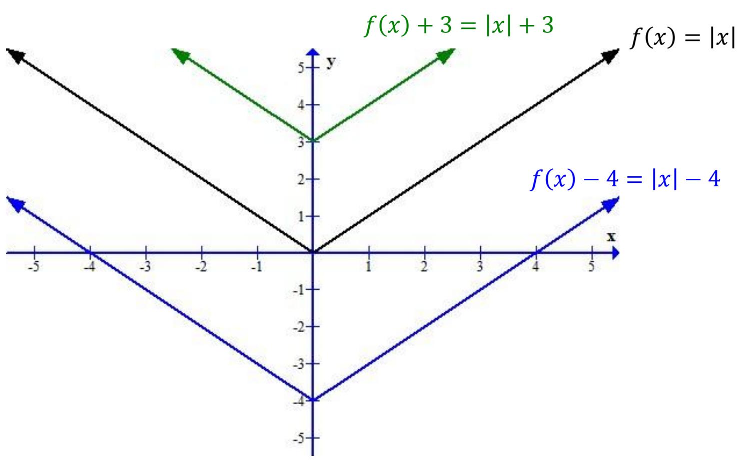
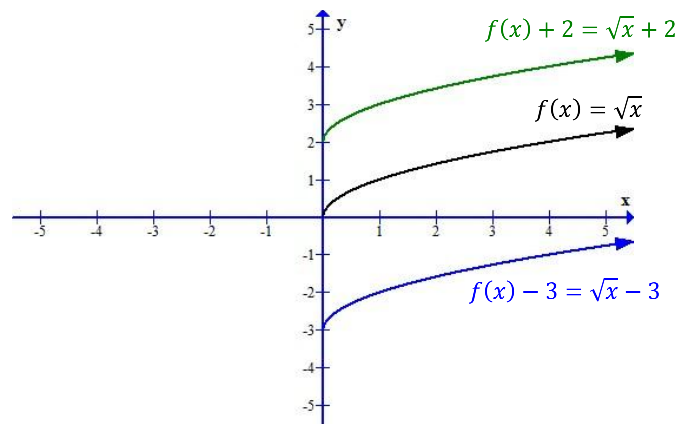
2. Graphical Transformations of Functions

In this section we will discuss how the graph of a function may be transformed either by shifting, stretching or compressing, or reflection. In this section let c be a positive real number.

Vertical Translations

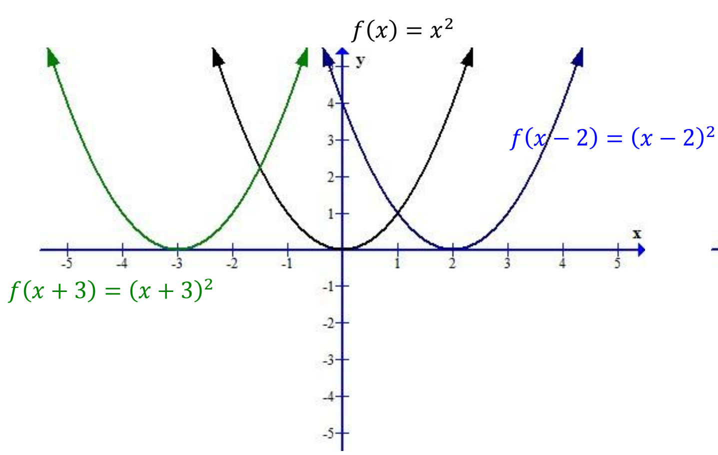
A shift may be referred to as a translation. If c is added to the function, where the function becomes $y = f(x) + c$, then the graph of $f(x)$ will vertically shift upward by c units. If c is subtracted from the function, where the function becomes $y = f(x) - c$, then the graph of $f(x)$ will vertically shift downward by c units. In general, a vertical translation means that every point (x, y) on the graph of $f(x)$ is transformed to (x, y + c) or (x, y – c) on the graphs of $y = f(x) + c$ or $y = f(x) - c$ respectively.

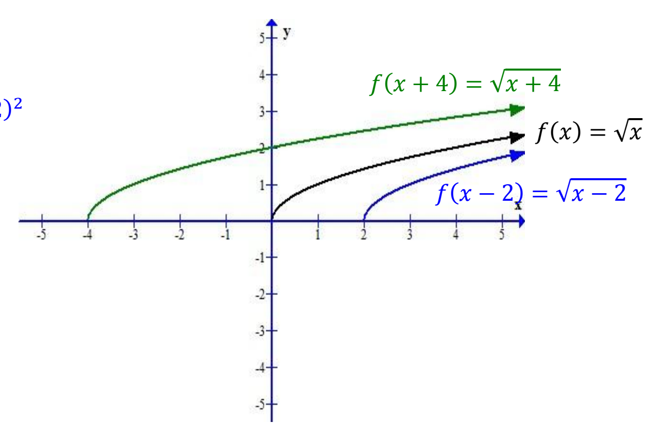




Horizontal Translations

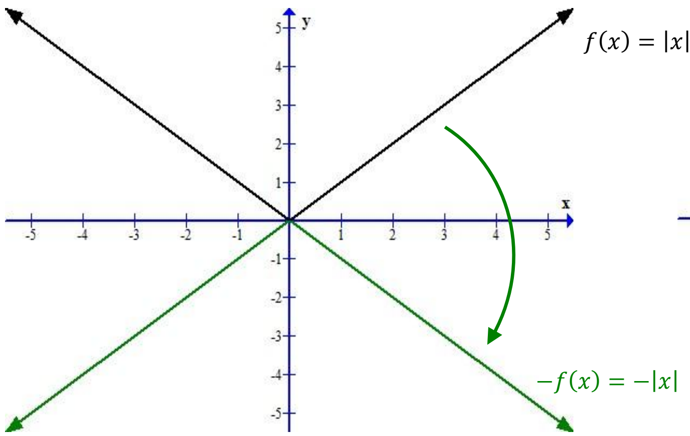
If c is added to the variable of the function, where the function becomes $y = f(x + c)$, then the graph of $f(x)$ will horizontally shift to the left c units. If c is subtracted from the variable of the function, where the function becomes $y = f(x - c)$, then the graph of $f(x)$ will horizontally shift to the right c units. In general, a horizontal translation means that every point (x, y) on the graph of $f(x)$ is transformed to (x – c, y) or (x + c, y) on the graphs of $y = f(x + c)$ or $y = f(x - c)$ respectively.

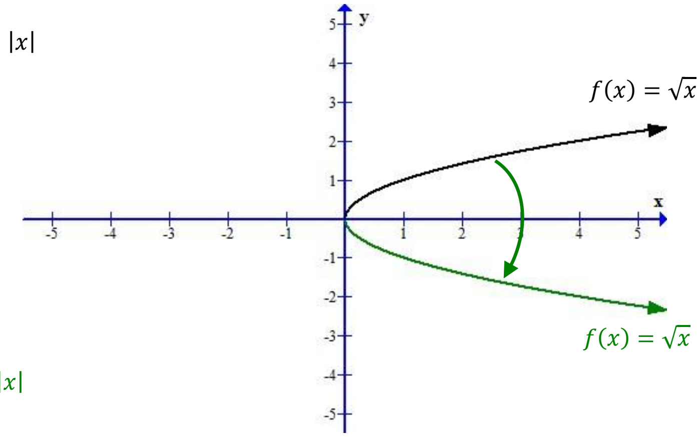




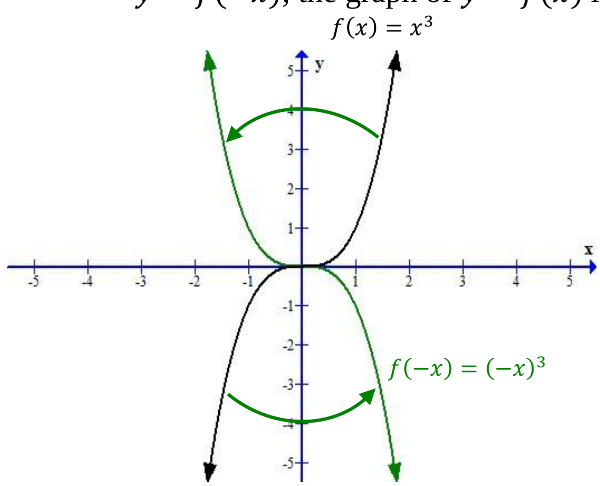
Reflection

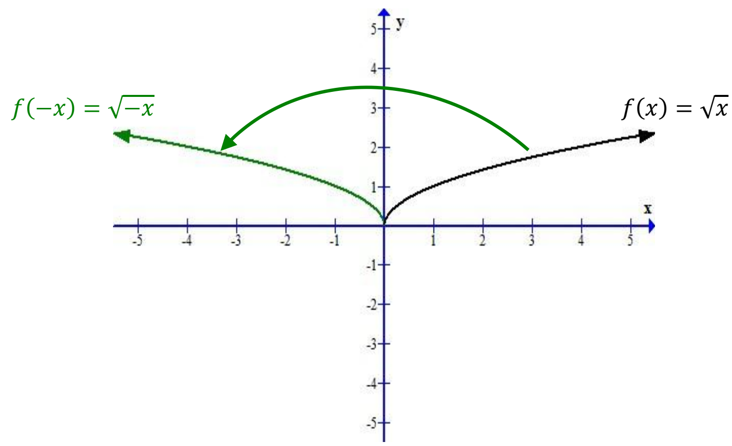
If the function or the variable of the function is multiplied by -1, the graph of the function will undergo a reflection. When the function is multiplied by -1 where $y = f(x)$ becomes $y = -f(x)$, the graph of $y = f(x)$ is reflected across the x-axis.





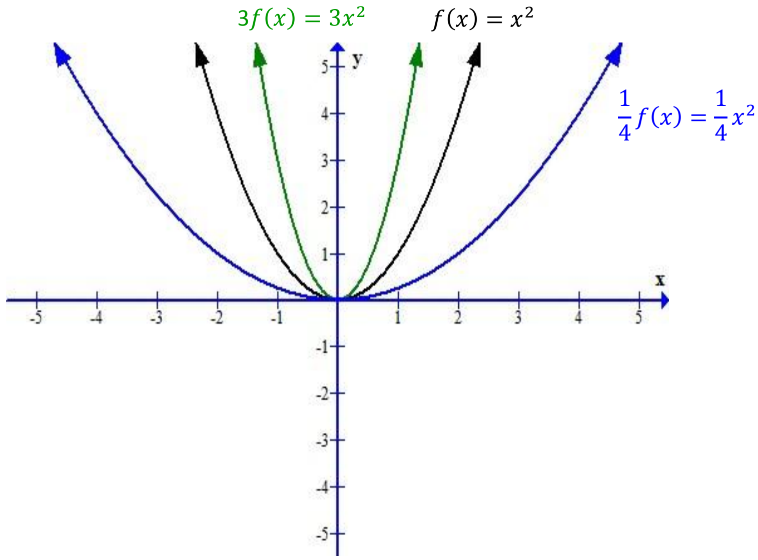
On the other hand, if the variable is multiplied by -1, where $y = f(x)$ becomes $y = f(-x)$, the graph of $y = f(x)$ is reflected across the y-axis.





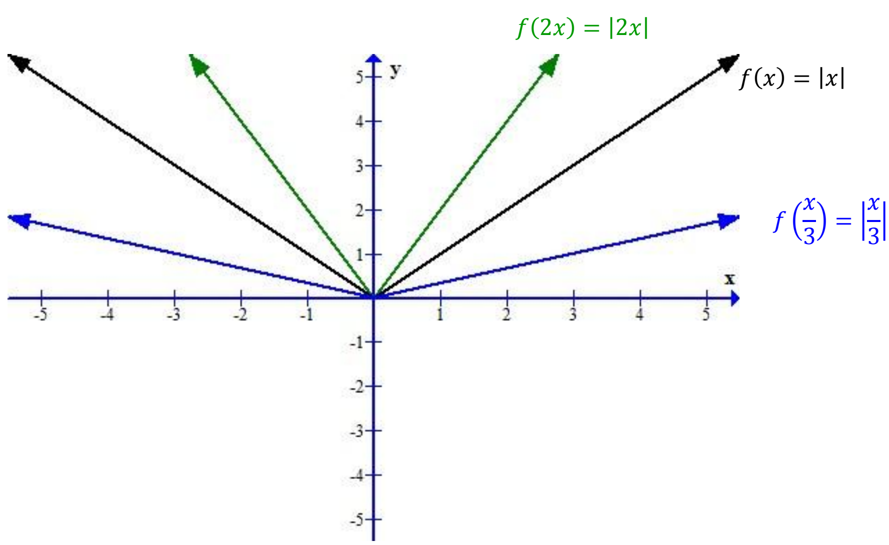
Vertical Stretching and Shrinking

If c is multiplied to the function then the graph of the function will undergo a vertical stretching or compression. So when the function becomes $y = cf(x)$ and $0 < c < 1$, a vertical shrinking of the graph of $y = f(x)$ will occur. Graphically, a vertical shrinking pulls the graph of $y = f(x)$ toward the x-axis. When $c > 1$ in the function $y = cf(x)$, a vertical stretching of the graph of $y = f(x)$ will occur. A vertical stretching pushes the graph of $y = f(x)$ away from the x-axis. In general, a vertical stretching or shrinking means that every point (x, y) on the graph of $f(x)$ is transformed to (x, cy) on the graph of $y = cf(x)$.



Horizontal Stretching and Shrinking

If c is multiplied to the variable of the function then the graph of the function will undergo a horizontal stretching or compression. So when the function becomes $y = f(cx)$ and $0 < c < 1$, a horizontal stretching of the graph of $y = f(x)$ will occur. Graphically, a vertical stretching pulls the graph of $y = f(x)$ away from the y-axis. When $c > 1$ in the function $y = f(cx)$, a horizontal shrinking of the graph of $y = f(x)$ will occur. A horizontal shrinking pushes the graph of $y = f(x)$ toward the y-axis. In general, a horizontal stretching or shrinking means that every point (x, y) on the graph of $f(x)$ is transformed to (x/c, y) on the graph of $y = f(cx)$.

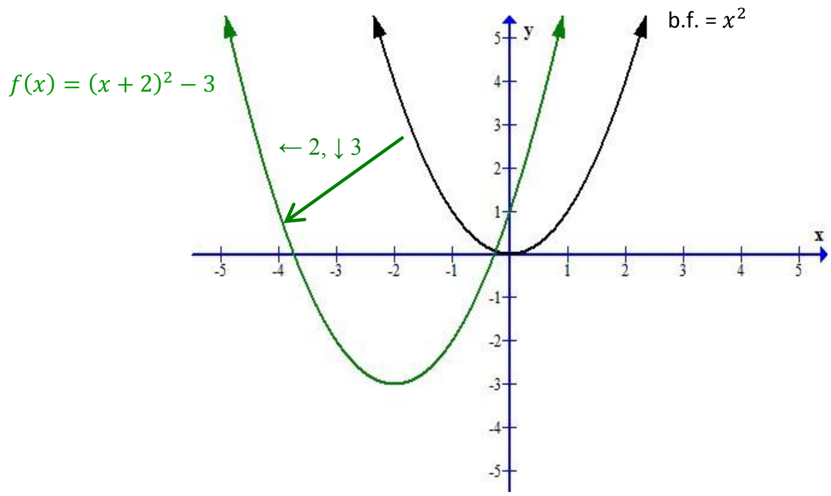


Transformations can be combined within the same function so that one graph can be shifted, stretched, and reflected. If a function contains more than one transformation it may be graphed using the following procedure:

Steps for Multiple Transformations
Use the following order to graph a function involving more than one transformation: 1. Horizontal Translation 2. Stretching or shrinking 3. Reflecting 4. Vertical Translation

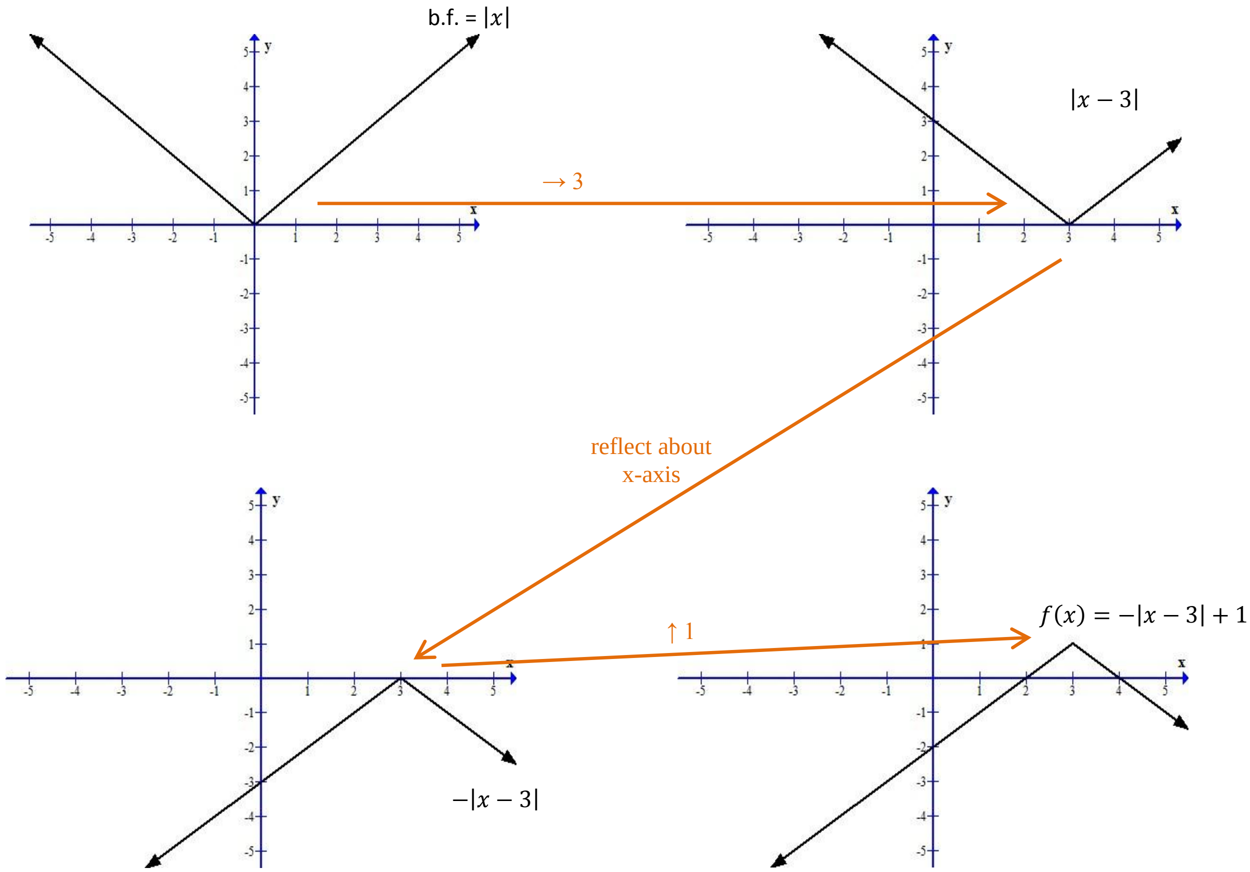
Examples: Graph the following functions and state their domain and range:

1. $f(x) = (x+2)^2 - 3$
   basic function (b.f.) = x^2, $\leftarrow 2, \downarrow 3$



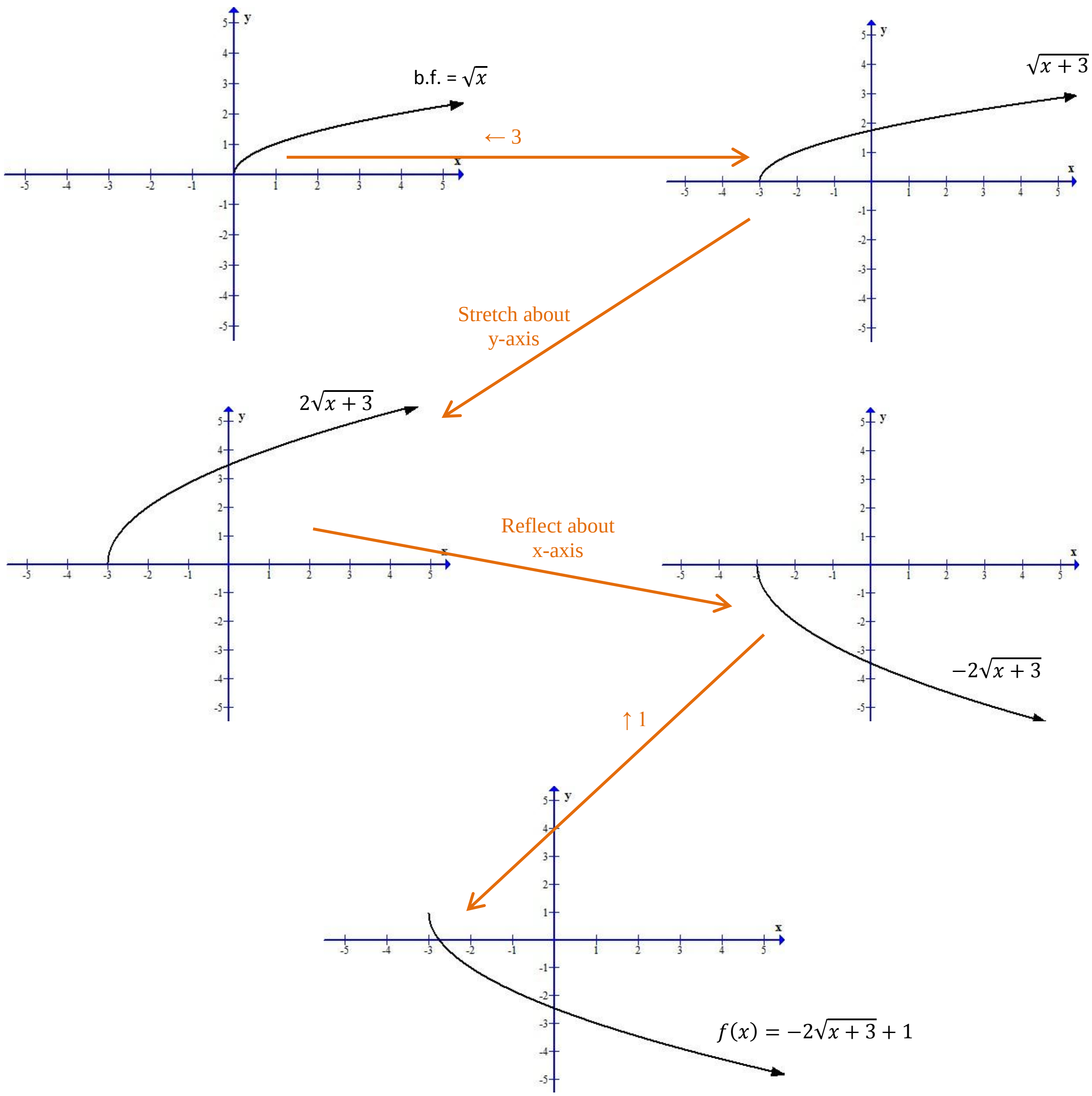
Domain = $(-\infty\ ,\ \infty)$
Range = $[-3\ ,\ \infty)$

2. $f(x) = -|x-3| + 1$
b.f. = $|x|$, $\rightarrow 3$, reflect about x-axis, $\uparrow 1$



Domain = $(-\infty\,,\ \infty)$
Range = $(-\infty\,,\ 1\,]$

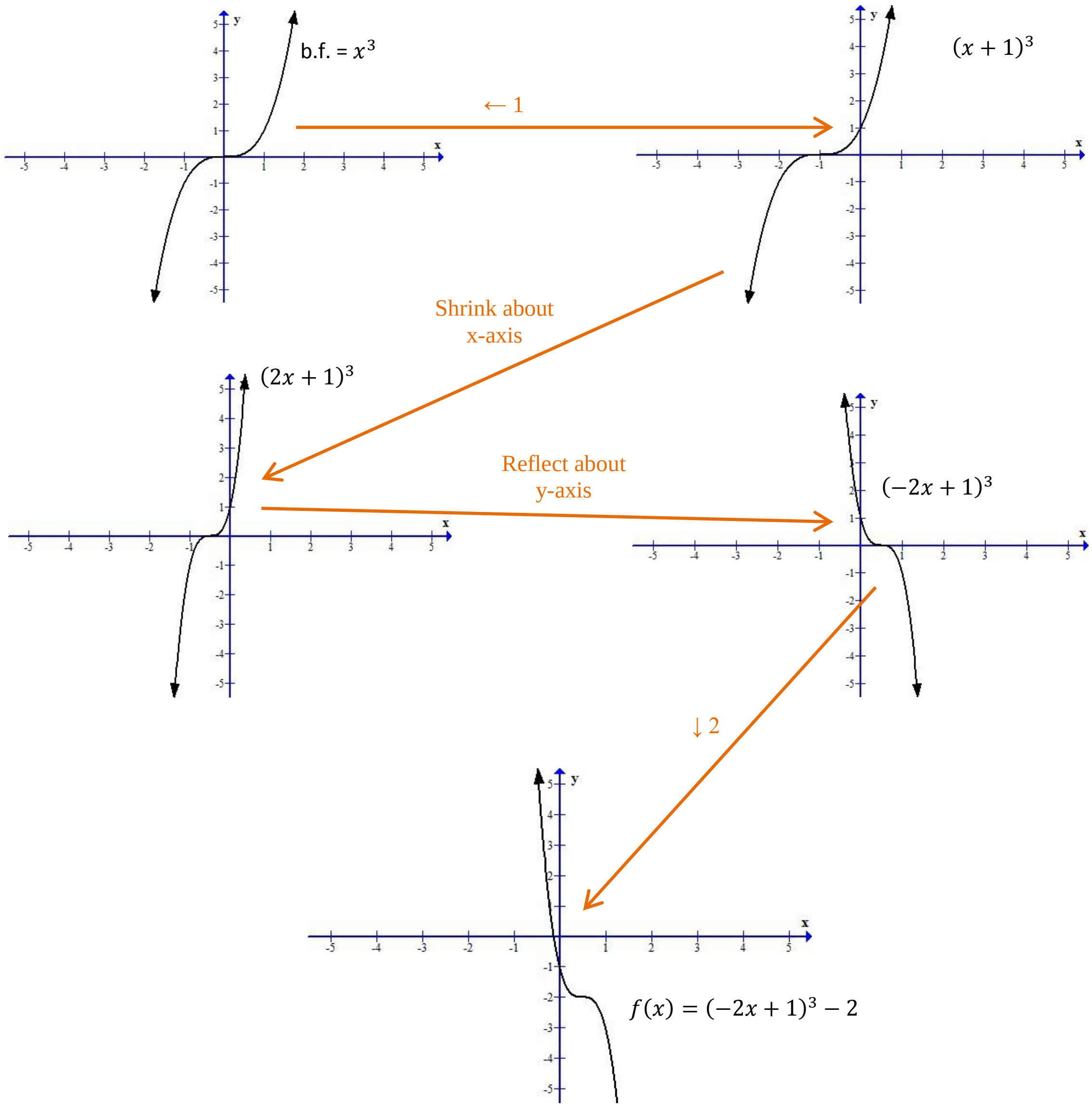
3. $f(x) = -2\sqrt{x+3} + 1$
b.f. = $\sqrt{x}$, ← 3, stretch about y-axis (c = 2), reflect about x-axis, ↑ 1



Domain =[-3 , ∞)
Range = ($-\infty$, 1]

4. $f(x) = (-2x + 1)^3 - 2$

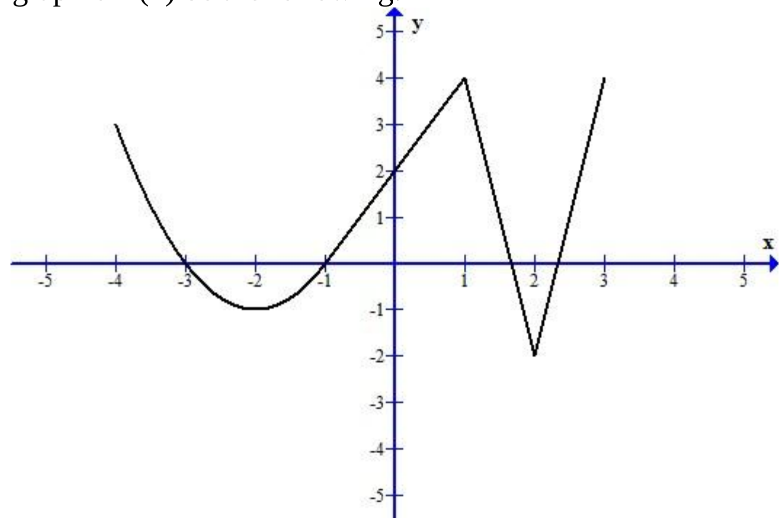
b.f. = x^3, ← 1, shrink about x-axis (c = 2), reflect about y-axis, ↓ 2



Domain = $(-\infty\,,\ \infty\,)$

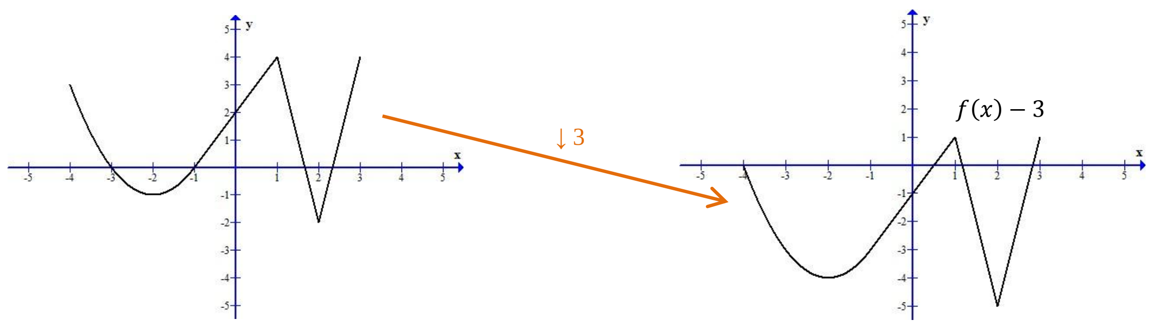
Range = $(-\infty\,,\ \infty\,)$

5. Let the graph of f(x) be the following:

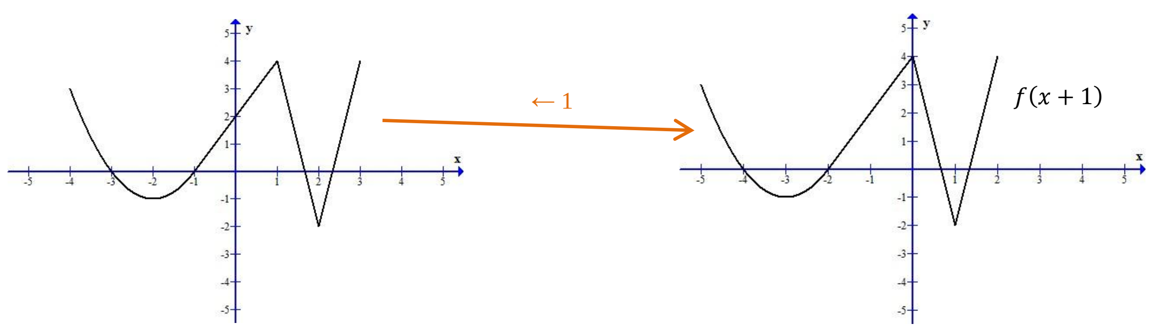


Graph the following problems:

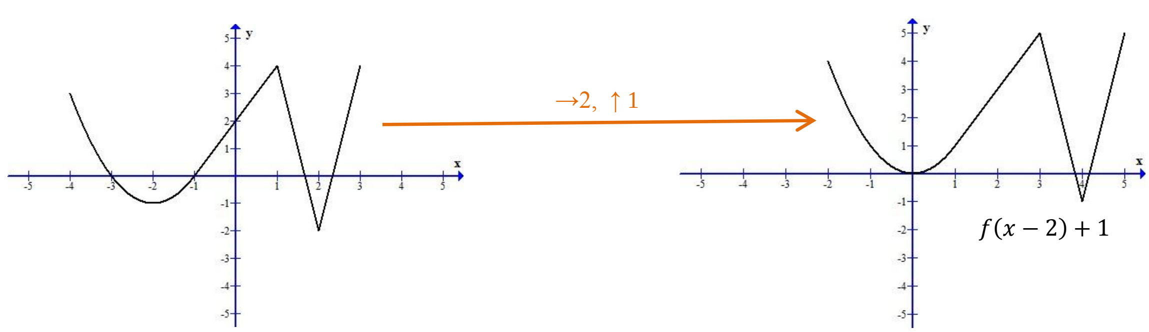
a. $f(x) - 3$



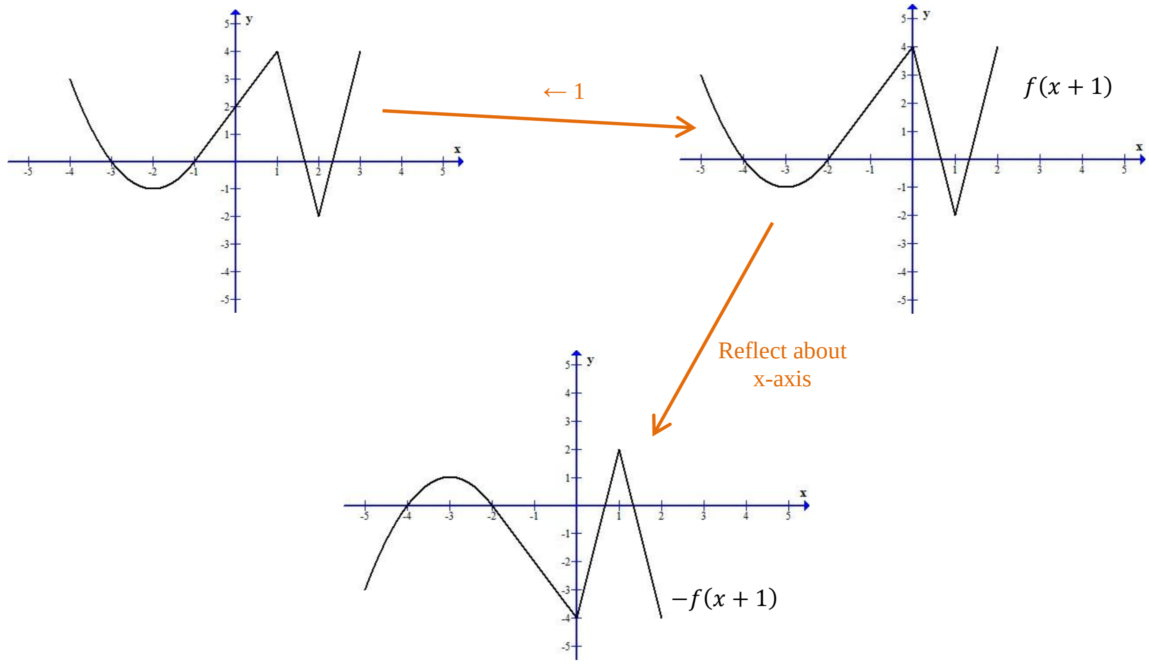
b. $f(x + 1)$



c. $f(x - 2) + 1$

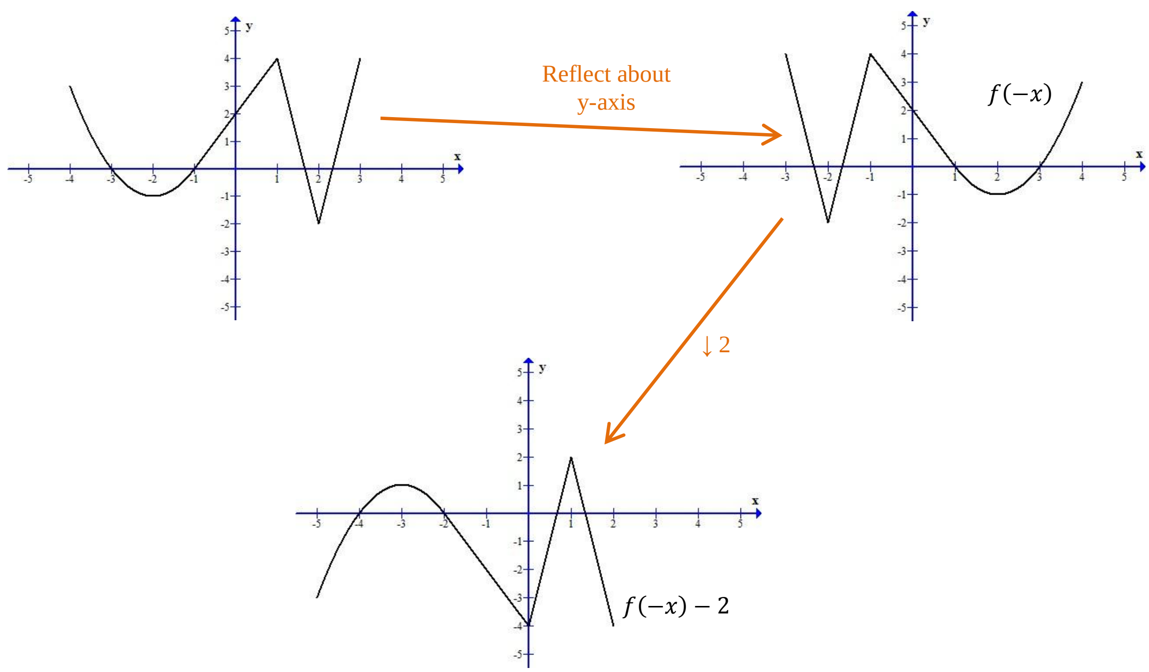


d. $-f(x+1)$



$\leftarrow 1$
$f(x+1)$
Reflect about x-axis
$-f(x+1)$

e. $f(-x)-2$



Reflect about y-axis
$f(-x)$
$\downarrow 2$
$f(-x)-2$

Transformations of the graphs of functions	
$f(x)+c$	shift $f(x)$ up c units
$f(x)-c$	shift $f(x)$ down c units
$f(x+c)$	shift $f(x)$ left c units
$f(x-c)$	shift $f(x)$ right c units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$cf(x)$	When $0<c<1$ – vertical shrinking of $f(x)$ When $c>1$ – vertical stretching of $f(x)$ **Multiply the y values by c**
$f(cx)$	When $0<c<1$ – horizontal stretching of $f(x)$ When $c>1$ – horizontal shrinking of $f(x)$ **Divide the x values by c**