Sec. 25 3-Space Hw (and some notes)

#1. An equation represents a plane in 3-space if it can be written as

$$ax + by + cz = d$$

(the classic coefficients here are a, b, c, d)

Additionally, at least one of a, b, c must be nonzero. This is so the plane doesn't reduce to a single point.

However, two of a, b, c may be zero and we'll still have a plane in 3-space. To wit:

If $a = 0$, $b = 0$, then $cz = d$ is a plane parallel to the xy-plane at height $z = d/c$ above the xy-plane.

Likewise, if $a = 0$, $c = 0$, then $by = d$ is a plane parallel to the xz-plane at height $y = d/b$ above the xz-plane.

You can do the third; i.e., $b = 0$, $c = 0$, $ax = d$, $x = d/a$, etc...

So, the examples of linear eqns of planes in 3-space are a, b, d, g, i, j, k

Looking at (i) for example:

$-x + y - z = 17$, $a = -1$, $b = 1$, $c = -1$, $d = 17$

And at (j): $y = 4$, $a = 0$, $b = 1$, $c = 0$, $d = 4$

#2 ~~Points (2, 0, t) and (u, 8, u) belong to~~ the plane $x + 2y + 3z + 4 = 0$

Find values of t + u.

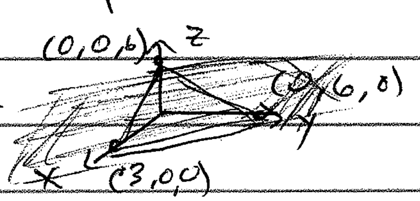
Substitute the pts' coordinates accordingly to produce a system of eqns:

$$2 + 2 \cdot 0 + 3t + 4 = 0 \rightarrow t = -2$$

$$u + 2(8) + 3u + 4 = 0 \longrightarrow u = -5$$

Thus, $(2, 0, -2)$ and $(-5, 8, -5)$ are pts in the plane.

#3 a) Find eqn of plane described by the points $(3,0,0)$, $(0,6,0)$ + $(0,0,6)$

[Sketch ]

Make a system of eqns:

From $px + qy + rz + s = 0$

$$\begin{cases} p(3) + q(0) + r(0) + s = 0 \\ p(0) + q(6) + r(0) + s = 0 \\ p(0) + q(0) + r(6) + s = 0 \end{cases}$$

Start eliminating terms the usual ways (solving for one variable in terms of another + substituting; elimination; matrix operations — you decide). Substitution is the obvious better method.

$$\begin{cases} 3p + s = 0 \\ 6q + s = 0 \\ 6r + s = 0 \end{cases}$$

$s = -6q = -6r$

$q = r$

$s = -3p = -6q \rightarrow \boxed{s = -3p}$

$\boxed{p = 2q} \rightarrow \boxed{p = 2r}$

$\boxed{q = r}$ ☆

$3p = 6r$

$\boxed{p = 2r}$ (same!)

$$\boxed{\begin{array}{l} p = 2r \\ q = r \\ s = -6r \\ r = r \end{array}}$$

We have all the unknowns in terms of r

(but you could have used another of q, r, s)

Substituting into $px + qy + rz + s = 0$

gives

$$2rx + ry + rz - 6r = 0$$

$$r(2x + y + z - 6) = 0$$

$$\boxed{2x + y + z = 6}$$

5)

#4