

Sec. 25 3-Space HW (and some notes)

#1. An equation represents a plane in 3-space if it can be written as

$$ax + by + cz = d$$

(the classic coefficients here are a, b, c, d)

Additionally, at least one of a, b, c must be nonzero. This is so the plane doesn't reduce to a single point.

However, two of a, b, c may be zero and we'll still have a plane in 3-space.

To wit:

If $a=0, b=0$, then $cz = d$ is a plane parallel to the xy -plane at height $z = d/c$ above the xy -plane.

Likewise, if $a=0, c=0$, then $by = d$ is a plane parallel to the xz -plane at height $y = d/b$ above the xz -plane.

You can do the third; i.e., $b=0, c=0$, $ax = d$, $x = d/a$, etc...

So, the ^{linear} examples of regions of planes in 3-space are a, b, d, g, i, j, k

Looking at (i) for example:

$$-x + y - z = 17, \quad a = -1, b = 1, c = -1, d = 17$$

And at (j): $y = 4, \quad a = 0, b = 1, c = 0, d = 4$

#2 — Points $(2, 0, t)$ and $(u, 8, u)$ belong to the plane $x + 2y + 3z + 4 = 0$

Find values of t + ~~u~~ .

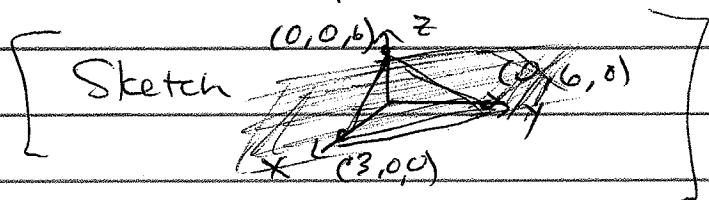
Substitute the pts' coordinates accordingly to produce a system of eqns:

$$2 + 2 \cdot 0 + 3t + 4 = 0 \rightarrow t = -2$$

$$u + 2(8) + 3u + 4 = 0 \rightarrow u = -5$$

Thus, ~~$(2, 0, -2)$~~ $(2, 0, -2)$ and $(-5, 8, -5)$ are pts in the plane.

#3 a) Find eqn of plane described by the points $(3, 0, 0), (0, 6, 0)$ + $(0, 0, 6)$



Make a system of eqns:

$$\text{From } px + qy + rz + s = 0$$

$$\begin{cases} p(3) + q(0) + r(0) + s = 0 \\ p(0) + q(6) + r(0) + s = 0 \\ p(0) + q(0) + r(6) + s = 0 \end{cases}$$

Start eliminating terms the usual ways
(solving for one variable in terms of another
+ substituting; elimination; matrix operations
— you decide). Substitution is the obvious
better method.

$$\begin{array}{l} s = -6q = -6r \\ q = r \end{array} \left\{ \begin{array}{l} 3p + s = 0 \\ 6q + s = 0 \\ 6r + s = 0 \end{array} \right. \begin{array}{l} s = -3p = -6q \rightarrow \boxed{s = -3p} \\ \boxed{p = 2q} \rightarrow \boxed{p = 2r} \\ \boxed{q = r} \rightarrow \star \\ 3p = 6r \\ \boxed{p = 2r} \text{ (same!)} \end{array}$$

$$\boxed{\begin{array}{l} p = 2r \\ q = r \\ s = -6r \\ r = r \end{array}}$$

We have all the unknowns in terms of r
(but you could have used another of q, r, s)

Substituting into $px + qy + rz + s = 0$

gives

$$\begin{aligned} & 2rx + ry + rz - 6r = 0 \\ & r(2x + y + z - 6) = 0 \\ & \boxed{2x + y + z = 6} \end{aligned}$$

B)

Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$$\begin{aligned} (1) \quad & 1 \cdot \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} + (-2) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \\ (2) \quad & 2 \cdot \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} + (-3) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 6 \\ 4 & 6 & 8 \\ 6 & 8 & 10 \end{bmatrix} \\ (3) \quad & 3 \cdot \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} + (-4) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 9 \\ 6 & 9 & 12 \\ 8 & 12 & 16 \end{bmatrix} \end{aligned}$$

After these operations, the matrix is in row echelon form. The next step is to use back substitution to find the inverse matrix. The operations performed are:

$$\begin{aligned} R_1 & \leftarrow R_1 - 2R_2 + R_3 \\ R_2 & \leftarrow R_2 - 3R_3 \\ R_3 & \leftarrow R_3 - 4R_2 \end{aligned}$$

#4

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

The inverse of the matrix is found by using the adjoint method. The adjoint of the matrix is the transpose of the cofactor matrix.

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 1(15-12) - 2(10-12) + 3(10-9) = 3 + 4 + 3 = 10 \\ A^{-1} &= \frac{1}{10} \begin{bmatrix} 15-12 & 10-12 & 10-9 \\ 12-15 & 5-9 & 5-6 \\ 10-9 & 5-6 & 4-6 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 3 & -2 & 1 \\ -3 & -4 & -1 \\ 1 & -1 & -2 \end{bmatrix} \end{aligned}$$