Ex 28.3

$f_x = 3y - 2xy - y^2$

$0 = y(3-2x-y)$ ①

$y=0$ or $3-2x-y=0$

$f_y = 3x - x^2 - 2xy$

$0 = x(3-x-2y)$ ②

$x=0$ or $3-x-2y=0$

Case 1 You have a candidate $(x_0, y_0) = (0,0)$

Substitute into f_x and f_y to see if ~~they~~ it satisfies the condition $f_x = 0$ AND $f_y = 0$.

A good way to ~~organize~~ express this is by the notation:

$$\left. f_x \right|_{(0,0)} = 3(0) - 2(0)(0) - 0^2 = 0$$

$$\left. f_y \right|_{(0,0)} = 3(0) - 0^2 - 2(0)(0) = 0$$

✓

$(0,0)$ is a crit. pt.

Case 2: $\underset{\text{from ①}}{y=0}$ and $\underset{\text{from ②}}{3-x-2y=0}$

Notice that you don't look at ① for both factors being zero. You mix ① + ② up!

Substitute $y=0$ into ②

$y = 0, \quad 3 - x - 2(0) = 0 \rightarrow x = 3$

You have a candidate $(3,0)$. Check it:

$f_x \Big|_{(3,0)} = 3(0) - 2(3)(0) - 0^2 = 0$ ✓

$f_y \Big|_{(3,0)} = 3(3) - 3^2 - 2(3)(0) = 9 - 9 - 0 = 0$ ✓

So $(3,0)$ is a crit. pt.

<u>Case 3</u> from ① $3 - 2x - y = 0$ and from ② $x = 0$

Substitute $x = 0$ into ①

$3 - 2(0) - y = 0, \quad y = 3$

You have a candidate $(0,3)$. Check it.

$f_x \Big|_{(0,3)} = 3(3) - 2(0)(3) - 3^2 = 9 - 0 - 9 = 0$ ✓

$f_y \Big|_{(0,3)} = 3(0) - 0^2 - 2(0)(3) = 0$ ✓

So $(0,3)$ is a critical pt

<u>Case 4</u> from ① $3 - 2x - y = 0$ & from ② $3 - x - 2y = 0$

Elimination or substitution works

~~3A2xHU~~

① $y = 3 - 2x$ into ② $3 - x - 2(3-2x) = 0$

$\longrightarrow 3 - x - 6 + 4x = 0 \rightarrow -3 + 3x = 0$

$\rightarrow x = 1$

Sub. into ① $y = 3 - 2(1) = 1$ $y = 1$

$(1,1)$ is last candidate. Check it:

$$f_x\Big|_{(1,1)} = 3(1) - 2(1)(1) - 1^2 = 3 - 2 - 1 = 0 \checkmark$$

$$f_y\Big|_{(1,1)} = 3(1) - 1^2 - 2(1)(1) = 3 - 1 - 2 = 0 \checkmark$$

So $(1,1)$ is a crit. pt.

Now set up $D = f_{xx} f_{yy} - f_{xy}^2$

& use criteria seen in class

$D > 0$ & f_{xx} (or f_{yy}) $> 0 \rightarrow (x_0, y_0)$ local min

$D > 0$ & f_{xx} (or f_{yy}) $< 0 \rightarrow (x_0, y_0)$ local max

$D < 0 \longrightarrow (x_0, y_0)$ saddle pt.

$D = 0 \longrightarrow$ no conclusion

To determine D quickly, you could find all the partials at each crit. pt. first.

~~(0,0)~~: $f_{xx} = -2y \qquad f_{yy} = -2x \qquad f_{xy} = 3-2x-2y = f_{yx}$

$f_{xx}(0,0) = 0 \qquad f_{yy}(0,0) = 0 \qquad f_{xy}(0,0) = 3$

$D = 0 \cdot 0 - 3^2 = -9 < 0$

so $\boxed{(0,0) \text{ is a saddle pt.}}$

Now do the others: $D =$

$(0,3)$ $\quad D = 0 - (-3)^2 = -9 < 0$

$\boxed{(0,3) \text{ is a saddle pt}}$

$(3,0)$ $\quad D = 0 - 9^2 < 0$

$\boxed{(3,0) \text{ saddle pt}}$

$(1,1)$ $\quad D = (-2)(-2) - (3-2-2)^2 = 4-1 > 0$

Check f_{xx} at $(1,1)$. $\rightarrow -2(1) = -2 < 0$

$D > 0,\ f_{xx} < 0 \rightarrow \boxed{(1,1) \text{ local max}}$