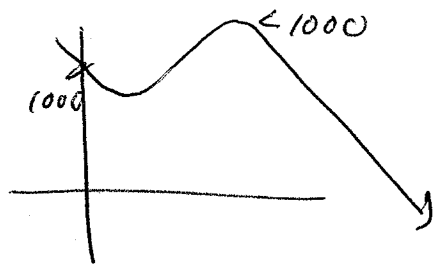
p 192 #9

$V(t) = -\frac{1}{3}t^3 + 8t^2 - 60t + 1000$ on $[0, 20]$

is a stock value fcn. wrt time.

$V(0) = 1000$ ~~(ass~~ (as given; Draco bought $1000 worth, so "time zero" value is 1000)

This fcn has general shape ↑ ↘ at the ends, so we know it's likely the max value is at $t = 0$. But maybe between 0 & 20 years, it rose above 1000. Like



~~Take the~~ Find out by ~~finding~~ crit pts + evaluating there.

$V'(t) = -t^2 + 16t - 60 = 0$

or $t^2 - 16t + 60 = 0$

Which factors as $(t-6)(t-10) = 0$, so at $t = 6$ & 10

there ~~is~~ are local extremes.

Recall that the second derivative test tells us whether the fcn is concave up (has a min) or down (has a max) at a crit. pt.

$\boxed{V''(t) = -2t + 16}$, $\quad V''(6) = -12 + 16 > 0$

c. up, $t = 6$ is local min

$V''(10) = -2(10) + 16 = -4 < 0$

c. down, $t = 10$ is local max

Compare $V(0)$ with $V(10)$ to see which is greater:

$N(0) = 1000$

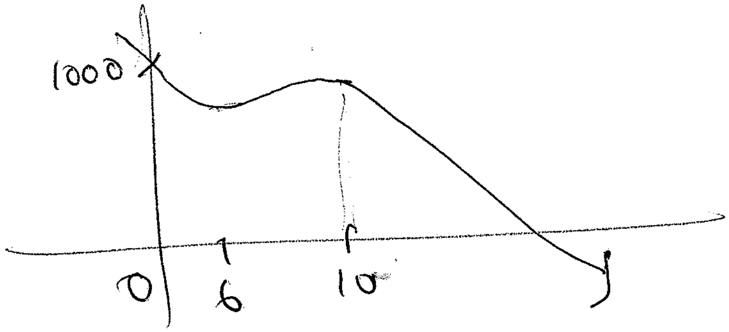
$$V(10) = -\tfrac{1}{3}(10^3) + 8(10^2) - 60(10) + 1000$$

$$= -333.33 + 800 - 600 + 1000$$

$$= -933.33 + 1800$$

$$= \$860 < 1000$$

So $V(0)$ is the max value (i.e., \$1000 at $t=0$)



$\boxed{p.\ 199\ \#1\ a-c,\ \#2}$

#1.a) $q = 60 - p$, $0 \le p \le 60$

$$E(p) = -\frac{p}{q} \cdot \frac{dq}{dp} = \left(\frac{-p}{\underbrace{60-p}_{q}}\right)(-1)$$

$$\boxed{E(p) = \frac{p}{60-p}}$$

b) $E(20) = \frac{20}{40} = \frac{1}{2} < 1$; At $p = 20$,

demand is inelastic ; price increase results in revenue increase because demand is not significantly affected, does not decrease.

c) $E(p) = 1$ at $p = ?$

$$\frac{+p}{60-p} = 1 \longrightarrow +p = 60 - p$$

$$2p = 60$$

At unit elasticity, neither raising nor lowering price affects revenue. $\longleftarrow$ $\boxed{p = 30}$

#2 $p = \dfrac{30}{q^{2.1}} = \dfrac{30}{q^{21/10}}$ price as fcn of demand

$p(q) \to q(p)$

cross $\otimes$

$\left(q^{\cancel{21/10}}\right)^{\cancel{10/21}} = \left(\dfrac{30}{p}\right)^{10/21} \longrightarrow q^{1} = \left(\dfrac{30}{p}\right)^{10/21} = \dfrac{30^{10/21}}{p^{10/21}}$

$\dfrac{1}{a} = a^{-1}$

$\rightarrow \boxed{q = 30^{10/21} \cdot p^{-10/21}} \Rightarrow -\frac{10}{21} - 1$

$\dfrac{dq}{dp} = 30^{10/21} \cdot \left(-\dfrac{10}{21}\right) p^{-31/21} = \dfrac{-30^{10/21}}{p^{31/21}} \cdot \dfrac{10}{21}$

$\rightarrow \boxed{E(p) = -\dfrac{p}{q} \cdot \dfrac{dq}{dp}} = \dfrac{-p}{\cancel{30^{10/21}}\, p^{-10/21}} \cdot \dfrac{-\cancel{30^{10/21}}}{p^{31/21}} \cdot \dfrac{10}{21}$

$= \dfrac{\cancel{p} \cdot \cancel{p^{10/21}} \cdot 10}{\cancel{p^{31/21}} \cdot 21} = \boxed{\dfrac{10}{21}}$ Notice that $E(p) < 1$ for any p.

Hence, $E(4) = \frac{10}{21} < 1$; demand is inelastic at any price.

Thus, any increase in price results in an increase in revenue. (Product is probably a necessity)

$\boxed{p. 206}$

#1 A linear eqn. in 3-space (i.e., the eqn. of a plane) is of the form (or can be put in the form)

$$\boxed{px + qy + rz + s = 0}$$

where not all p, q, r are zeros. (But two of them could be zero such as when $p, q = 0$, so (coeffs p, q, r)

$rz = -s$, or $z = -s/r$, the plane through $z = \frac{-s}{r}$, parallel to the xy-plane.)

Thus, #1a, b, d, e, i, j, and k all are linear eqns in 3-space.

$\boxed{\#3a}$ Take each ordered triple an substitute into $px + qy + rz + s = 0$, three eqns.

Given

$$\boxed{\begin{matrix}(3,0,0)\\(0,6,0)\\(0,0,6)\end{matrix}} \quad \begin{aligned} 3p + 0q + 0r + s &= 0 \\ 0p + 6q + 0r + s &= 0 \\ 0p + 0q + 6r + s &= 0 \end{aligned}$$

Simplify $\left\{ \boxed{\begin{aligned} 3p &= -s \\ 6q &= -s \\ 6r &= -s \end{aligned}} \right.$

Object - name all 4 variables p, q, r, s in terms of only one.

Put q, r, s in terms of p by simple substitutions -

$\boxed{s = -3p}$, $s = -6q \implies -3p = -6q \implies \boxed{q = \frac{p}{2}}$

$6r = -s = 3p \implies \boxed{r = \frac{p}{2}}$

Substitute all these p terms into $px + qy + rz + s = 0$

$$px + \frac{p}{2}y + \frac{p}{2}z - 3p = 0$$

Factor out p: $p\left(x + \frac{y}{2} + \frac{z}{2} - 3\right) = 0$

Divide by p: $x + \frac{y}{2} + \frac{z}{2} - 3 = 0$
(it disappears)

Which is equivalent to : $\boxed{2x + y + z - 6 = 0}$
(multiplying by 2)

or

$\boxed{2x + y + z = 6}$

$\boxed{\#3a)}$

Alternate - put all in terms of s

$\cancel{s = 3p}$

$p = -s/3$
$q = -s/6$
$r = -s/6$

$$\boxed{px + qy + rz + s = 0}$$

General form of plane

$$-\frac{s}{3}x - \frac{s}{6}y - \frac{s}{6}z + s = 0$$

$$-\cancel{s}\left(\frac{x}{3} + \frac{y}{6} + \frac{z}{6} - 1\right) = 0$$

X6

LCD

$$2x + y + z - 6 = 0$$

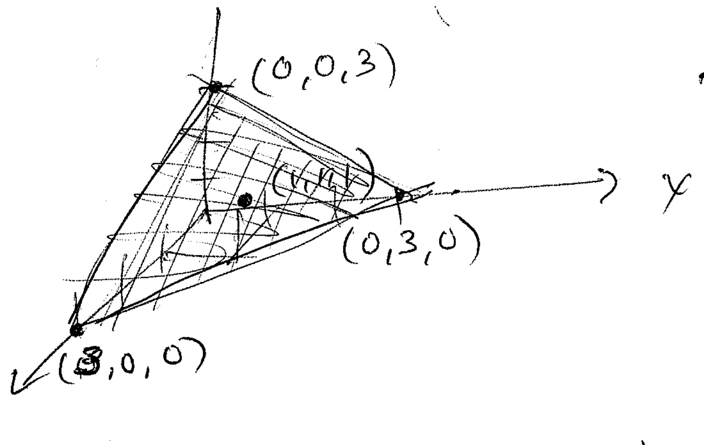
$$\boxed{2x + y + z = 6}$$

$\boxed{\#4a}$ Plane $x + y + z = 3$

- x-int (let $y = z = 0$): $x = 3$ — $(3,0,0)$ x-intercept

☆

- y-int (let $x = z = 0$): $y = 3$ — $(0,3,0)$ y-intercept
- z-int (let $y = x = 0$) $z = 3$ — $(0,0,3)$ z-intercept



$x = y = 1$ at $z = 1$

from simple sub:

$$1 + 1 + z = 3$$
$$2 + z = 3$$
$$z = 1$$

Hence, $(1,1,1)$ is on the plane

Aside ~~Eta~~ For the eqn of a plane

$$px + qy + rz + s = 0$$

where p & $q = 0$, $r = 3$, $s = 6$

draw the plane, that is, $3z + 6 = 0$

Label any two points on this plane.

Describe the plane in words.

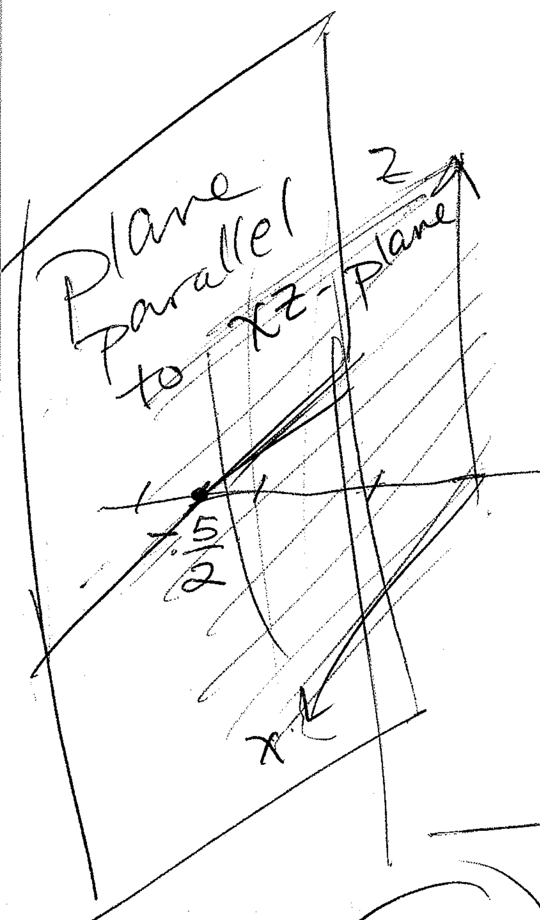
Ex1 $\cancel{p}x + qy + \cancel{r}z + s = 0$

& $p, r = 0$

$qy + s = 0$ in fact, let $q = 2$, $s = 5$

$2y + 5 = 0$ $\boxed{y = -5/2}$

for all x & z



Ex2 $3x + 1 = 0$ in 3-space

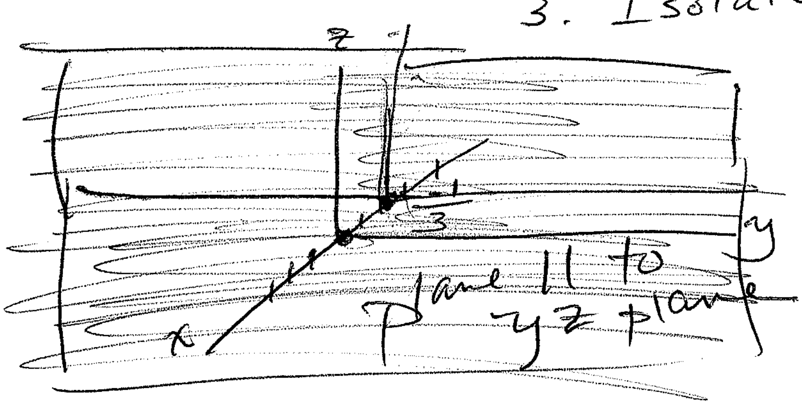
1. Is this linear? yes!

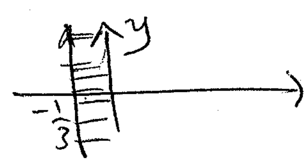
$px + qy + rz + s = 0$

2. $q = 0$ & $r = 0$

3. Isolate $3x = -1$, $\boxed{x = -\frac{1}{3}}$

for all y + z

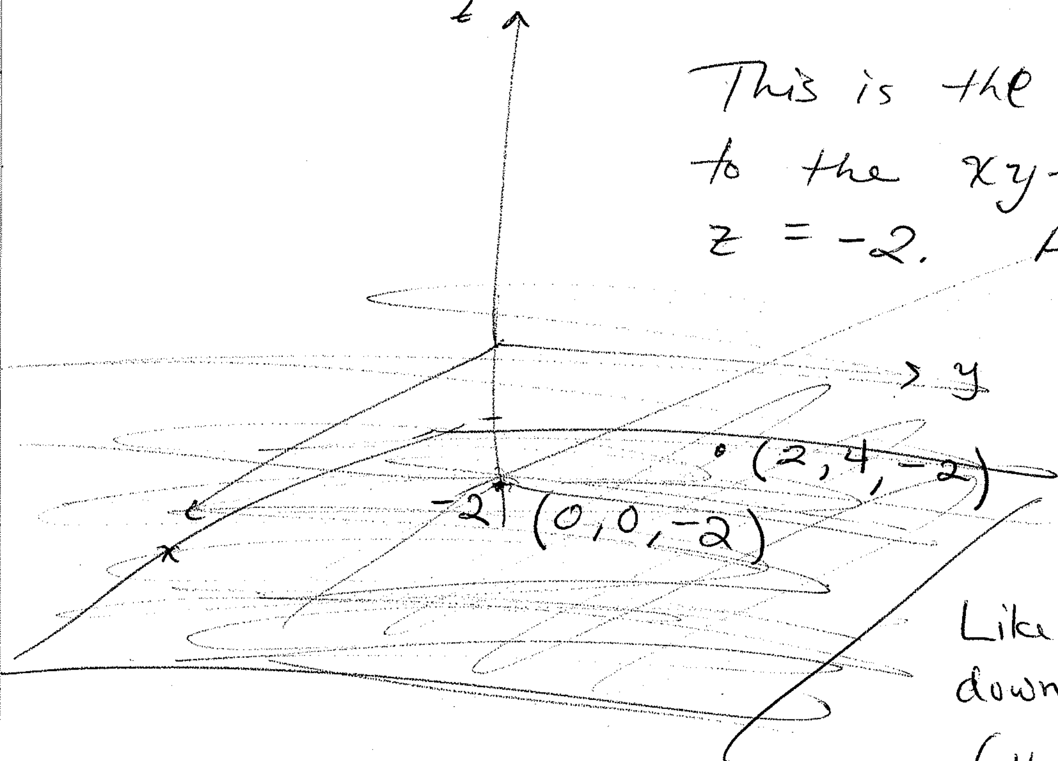




$$3z + 6 = 0 \longrightarrow z = -\frac{6}{3} = -2$$

So $z = -2$ for all pairs (x, y, z)

This is the plane parallel to the xy-plane through $z = -2$. All ordered triples look like $(x, y, -2)$



Like a floor 2 feet down from a false floor (the xy-plane)

#3b Same process:

$(1,2,1)$: $1p + 2q + 1r + s = 0$ ①
$(0,0,3)$: $0p + 0q + 3r + s = 0$ ②
$(2,0,2)$: $2p + 0q + 2r + s = 0$ ③

$$\left.\begin{array}{ll} ① & p + 2q + r = -s \\ ② & 3r = -s \end{array}\right\} \quad p + 2q + r = 3r \quad \boxed{p + 2q - 2r = 0}$$

$$\left.\begin{array}{ll} ③ & 2p + 2r = -s \\ ② & 3r = -s \end{array}\right\} \quad 2p + 2r = 3r \rightarrow \boxed{2p = r}$$

Keep substituting + simplifying till all variables are expressed in terms of one, say p.

$$p + 2q - 2r = p + 2q - 2(2p) = 0 \rightarrow 2q = 3p \quad \boxed{q = \tfrac{3}{2}p}$$

So far, we have $r = 2p$, $q = \frac{3}{2}p$

Since $-s = 3r$, $s = -3r = -3(2p) = \boxed{-6p = s}$

With s, q, r all in terms of p, substitute into general eqn. of plane:

$$px + qy + rz + s = 0$$
$$px + \tfrac{3}{2}py + 2pz - 6p = 0$$
$$\cancel{p}\left(x + \tfrac{3}{2}y + 2z - 6\right) = 0 \rightarrow x + \tfrac{3}{2}y + 2z = 6$$

$$\boxed{2x + 3y + 4z = 12}$$

~~[illegible]~~ $\boxed{\text{Sec 26-29}}$ $\boxed{\text{multivariate fcns}}$ Sec 26 is – mainly to

Sec 26 show you how to deal with $f(x,y)$, $C(x,y,z)$ to prepare for multivariate optimization (Sec 28 & 29). By Sec 28, you've learned partial differentiation, so you don't need to compute marginal production, cost, etc. by straight computation. The downside is you have to solve the system of partials in order to find the crit pts.

Sec 28 There's still the matter of interpreting your answer in those chapters; what you always need to do is, when you find an (x,y) you deem to be a crit. pt, you must check to see if ~~[illegible]~~ it yields a max, min, saddle, or inconclusive. (D tests)

When asked to evaluate at the crit pt, you just plug it into $f(x,y)$.

Sec 29 If you are asked to find an optimal level of whatever (x,y) is in question when constrained by another condition, turn to Lagrange multiplier method.

The format is this:

- $f(x,y)$ is the optimizing fcn (what you seek to maximize or minimize)
- $g(x,y)$ is formed from some constraint
  $ax + by = c$, which we set = zero
  & name $g(x,y)$
- $F(x,y,\lambda) = f(x,y) + \lambda g(x,y)$

  The new fcn. whose partials wrt x, y, & λ lead to the solution.

$\boxed{\text{p. 233}}$ – Let's do the optimization in multivariate cases, rather than revisit Sec 23. This is more germaine to the test. By the way, you could do some as we did in previous chapter, but we'll use Lagrange multipliers instead.

#[illegible]

I'm skipping Sec 28 for now, because I need you to see the connection to previous optimization method, where to get it down to one variable we substituted (remember area + perimeter problem?) rather than this current multivariable method.

p. 233 #2

a) Maximize $f(x,y) = 2xy$, subj to (constrained by)

$x + y = 12$

- Rewrite the constraint as a fcn.

$$g(x,y) = x + y - 12$$

- Form $F(x,y,\lambda) = \underbrace{f(x,y)}_{\text{optimiz. fcn.}} + \underbrace{\lambda g(x,y)}_{\text{Lambda multiplier with constraint}}$

$$F(x,y,\lambda) = 2xy + \lambda(x+y-12)$$

$$\boxed{F(x,y,\lambda) = 2xy + \lambda x + \lambda y - 12\lambda}$$

- Take all the partials; set = zero

$$\begin{cases} F_x = 2y + \lambda = 0 \\ F_y = 2x + \lambda = 0 \\ F_\lambda = x + y - 12 = 0 \end{cases}$$

Solve the system for x & y. (λ is the helper variable here; its partial is key to finding x & y)

$F_x: \; 2y + \lambda = 0 \quad \to \quad y = -\lambda/2$

$F_y: \; 2x + \lambda = 0 \quad \longrightarrow \quad x = -\lambda/2$

into F_λ: $\; -\lambda/2 - \lambda/2 - 12 = 0$

$$-\lambda = 12 \quad \longrightarrow \quad \boxed{\lambda = -12}$$

Sub into F_x, F_y to get x, y:

$2y - 12 = 0, \quad y = 6$

$2x - 12 = 0, \quad x = 6$

Thus, $f(6,6) = 2(6)(6) = 72$ is the maximum under the constraint.

Notice we don't check that this is a max, as done with D testing of previous chapter.

But it's not hard to check a pt. <u>near</u> $(6,6)$ to see if $f(x,y)$ is less there.

Say $x = 5, \; y = 7$ (since $x + y = 12$)

$f(5,7) = 2(35) = 70 < 72$ Voila!

$\boxed{\#2b}$ $\quad$ <u>Minimize</u> $\quad f(x,y) = x^2 + 3y^2$

subj. to $\quad x - y + 1 = 0$

$$F(x,y,\lambda) = x^2 + 3y^2 + \lambda(x - y + 1)$$

$$= x^2 + 3y^2 + \lambda x - \lambda y + \lambda$$

$$\boxed{F_x = 2x + \lambda = 0} \qquad \boxed{F_y = 6y - \lambda = 0}$$

$$\boxed{F_\lambda = x - y + 1 = 0}$$

Solve the system for λ first in terms of x & y. (Use F_x, F_y). Subsitute into F_λ to get λ. Resub. into F_x & F_y to get x & y.

$$\begin{aligned} 2x + \lambda &= 0 \longrightarrow \\ 6y - \lambda &= 0 \longrightarrow \end{aligned} \quad \boxed{\begin{aligned} x &= -\lambda/2 \\ y &= \lambda/6 \end{aligned}} \longrightarrow \text{into } F_\lambda$$

$$-\lambda/2 - \lambda/6 + 1 = \frac{-3\lambda - \lambda}{6} + 1 = 0$$

$$\longrightarrow \quad -4\lambda = -6 \longrightarrow \boxed{\lambda = +3/2}$$

Sub. into F_x, F_y: $\searrow$

$2x + 3/2 = 0 \longrightarrow$ $\boxed{x = -3/4}$

$6y - 3/2 = 0 \longrightarrow$ $\boxed{y = +1/4}$

Thus, $f(-3/4, 1/4) = 9/16 + 3/16 = 12/16$ or $3/4$

is the minimum value of f subj. to the constraint.

$\boxed{\#2d}$ $f(x, y) = x + y - x^2 - y^2$

subj to $x + 2y = 6$ $(g(x,y) = x + 2y -$

$$F(x, y, \lambda) = x + y - x^2 - y^2 + \lambda(x + 2y - 6)$$
$$= x + y - x^2 - y^2 + \lambda x + 2\lambda y - 6\lambda$$

$$F_x = 1 - 2x + \lambda = 0$$
$$F_y = 1 - 2y + 2\lambda = 0$$
$$F_\lambda = x + 2y - 6 = 0$$

Solve each for x, y in terms of λ

$\downarrow$

Substitute into F_λ and solve for λ

$\downarrow$

Resubstitute λ into F_x, F_y

F_x: $-2x = -\lambda - 1$

$$\boxed{x = \frac{\lambda + 1}{2}}$$

F_y: $-2y = -2\lambda - 1$

$$\boxed{y = \frac{2\lambda + 1}{2}}$$

F_λ: $$\frac{\lambda+1}{2} + \cancel{2}\left(\frac{2\lambda+1}{\cancel{2}}\right) - 6 = 0$$

$$\frac{\lambda+1}{2} + 2\lambda + 1 = 6$$

$$\lambda + 1 + 4\lambda + 2 = 12$$

$$5\lambda = 9$$

$$\boxed{\lambda = 9/5}$$

Into F_x, F_y: $x = \frac{9/5+1}{2} = \frac{14/5}{2} = \frac{14}{10}$

$y = \frac{2(9/5)+1}{2} = \frac{23/5}{2} = \frac{23}{10}$

$$f\left(\frac{14}{10}, \frac{23}{10}\right) = \frac{14}{10} + \frac{23}{10} - \left(\frac{14}{10}\right)^2 - \left(\frac{23}{10}\right)^2$$

$$= \frac{37}{10} - \frac{196}{100} - \frac{529}{100}$$

$$= \frac{370 - 196 - 529}{100} = \boxed{-\frac{355}{100}}$$

$$\begin{array}{r} 23 \\ \underline{23} \\ 69 \\ \underline{46} \\ 529 \end{array} \qquad \begin{array}{r} 529 \\ \underline{174} \\ 355 \end{array}$$