Math 220: Exam 3 Name ~~Key~~ McKenzie notes

Spring 2015 Instructor (not the official key)

Page	1	2	3	4	Total	Course Points
Points	18	30	30	22	100	150
Score						

- Calculators are not permitted for this test.
- Show your work unless the problem requires only a short answer.
- This test is single-sided. Use the backs of the sheets provided for scratch work, but write your answers in the space provided for each problem. No scrap paper is permitted on this exam.

1. (12 points) The quantity of items sold depends on the price, as expressed by the function $q(p) = \sqrt{100 - 4p^2}$.

a) Find the elasticity function $E(p)$.

$$E(p) = \frac{-p}{\sqrt{100-4p^2}} \cdot \frac{1}{2}(100-4p^2)^{-1/2} \cdot -8p = \boxed{\frac{4p^2}{100-4p^2}}$$

b) If the current price is \$3 per item, what would a slight increase in price do to revenue? Explain your answer.

$$E(3) = \frac{4 \cdot 9}{100 - 4 \cdot 9} = \frac{36}{64} < 1$$

Demand is inelastic, not responsive to price increase, revenue would increase

c) Find the price that should be charged to yield the maximum revenue.

$$E(p) = \frac{4p^2}{100-4p^2} = 1 \rightarrow 4p^2 = 100 - 4p^2$$

$$8p^2 = 100$$

$$p = \sqrt{\frac{100}{8}} = \frac{10}{2\sqrt{2}}$$

$$\boxed{p \approx \frac{\$5}{\sqrt{2}} \text{ or } \$3.20}$$

2. (6 points) Determine the absolute minimum and the absolute maximum for the function on the specified interval.

$h(t) = \sqrt[3]{t}\,(8 - t)$ on $[-1, 7]$

$$h'(t) = \frac{1}{3}t^{-2/3}(8-t) + t^{1/3}(-1) = \frac{8-t}{3t^{2/3}} - t^{1/3} = \frac{8-t-t}{3t^{2/3}}$$

$$= \frac{8-2t}{3t^{2/3}} = 0 \quad \text{when} \quad t = 4$$

abs min

$$h(4) = \sqrt[3]{4} \cdot 4 \qquad h(-1) = (-1)(9) = -9 \qquad h(7) = \sqrt[3]{7}\,(1)$$

$$4\sqrt[3]{4} > \sqrt[3]{7} \quad \text{so}$$

3. (30 points) Given $g(x) = x^3 - 6x^2 - 15x + 4$,

a) Find the y-intercept of g. State your answer as an ordered pair.

$g(0) = 4$ $\qquad \boxed{(0,4)}$

b) State all intervals where g is....

increasing: $\qquad g'(x) = 3x^2 - 12x - 15 = 3(x^2 - 4x - 5) = 3(x-5)(x+1)$

$\boxed{(-\infty, -2) \cup (6, \infty)}$ $\quad > 0 \quad$ $g'(x) = 0$ at $x = 5, -1$ $\quad$ −2 −1 0 5 6

decreasing:

$\boxed{(-2, 6)}$

concave upward: $\qquad g''(x) = 6x - 12 = 0$ at $\boxed{x = 2}$

$g''(x) > 0$ when $6x - 12 > 0$ or $x > 2$ $\qquad (2, \infty)$ c. up

concave downward:

$g''(x) < 0$ when $6x - 12 < 0$ or $x < 2$ $\qquad (-\infty, 2)$ c. down

c) Find all *ordered pairs* belonging to g representing....

local minima:

$g''(-1) = -6 - 12 = -18$, c.d., local max so $g(-1) = 12$ or $\boxed{(-1, 12) \text{ local max}}$

$g''(5) = 30 - 12 > 0$, c. up., local min at $\boxed{(5, -96)}$

local maxima:

$\boxed{(-1, 12)}$ from $g''(-1)$

inflection points:

$\boxed{(0, 2)}$ $\qquad g(2) = 8 - 24 - 30 + 4 = -42$

d) Sketch the graph of g on the axes provided on the last page of the exam (page 5). Scale your axes appropriately. Your graph should accurately reflect the information found in parts a) through c) above.

4. (30 points) Given $f(x) = \frac{x}{1+x^2}$ and $f'(x) = \frac{1-x^2}{(1+x^2)^2}$ and $f''(x) = \frac{2x(x^2-3)}{(1+x^2)^3}$, determine the following…

a) The domain of f. $x \in \mathbb{R}$ $\boxed{(-\infty, \infty)}$

b) ordered pairs representing the x and y intercepts on the graph of f.

$f(0) = 0$ $\boxed{(0,0) \text{ is } x + y \text{ intercept}}$

c) Equations of all asymptotes to the graph of f.

No V.A $\boxed{y = 0 \text{ is HA}}$

d) Intervals where f is increasing and where f is decreasing.

$f'(x) = 0$ when $x = \pm 1$ $f'(-2) = \frac{-}{+}$, $\boxed{f \downarrow \text{ on } (-\infty, -1)}$

$f'(0) = \frac{+}{+} > 0$, $\boxed{f \uparrow \text{ on } (-1, 1)}$, $f'(2) = \frac{-}{+}$, $\boxed{f \downarrow \text{ on } (1, \infty)}$

e) Ordered pairs of local extrema.

$f(-1) = \frac{-1}{2}$ $(-1, -\frac{1}{2})$ local min

$f(1) = \frac{1}{2}$ $(1, \frac{1}{2})$ local max

f) Intervals where f is concave upward and where f is concave downward.

c.d.   c.u.   c.d.

$-\sqrt{3}$ 0 $\sqrt{3}$

$f''(x) = 2x(x^2-3) = 0$ at $\boxed{x = 0, \pm\sqrt{3}}$

$f''(-2) = \frac{-}{+} < 0$, $\boxed{\text{c.d. on } (-\infty, -\sqrt{3}) + (\sqrt{3}, \infty)}$

g) Ordered pairs of inflection points.

$f''(-1) = \frac{+}{+} > 0$, c. up

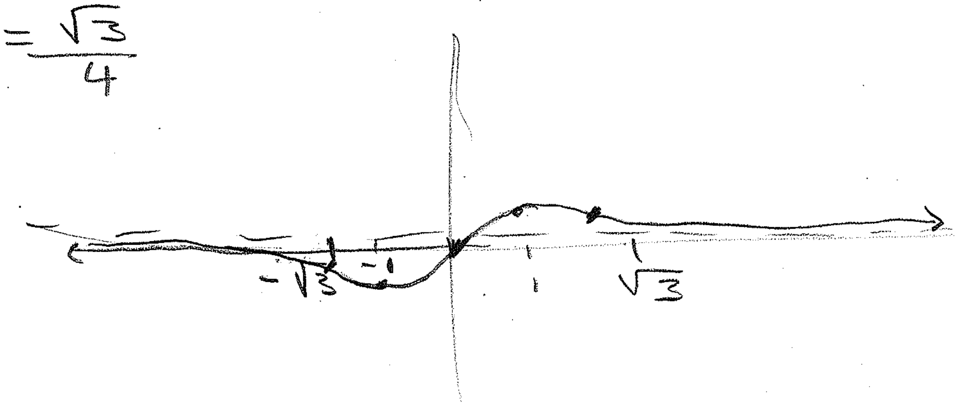
$f''(1) = \frac{-}{+} < 0$ c. down

$f(0) = 0$ $(0,0)$

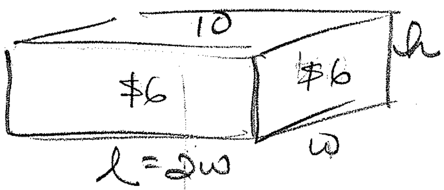
$f(-\sqrt{3}) = \frac{-\sqrt{3}}{4}$, $(-\sqrt{3}, \frac{-\sqrt{3}}{4})$

h) Sketch the graph of f on the axes provided on the last page of the exam (page 5). Scale your axes appropriately. Your graph should accurately reflect the information found in parts a) through g) above..

$f(\sqrt{3}) = \frac{\sqrt{3}}{4}$



5. (10 points) A box is made from two different types of material. The cheaper material is used for the sides of the box and costs $6 per square foot. The more expensive material is used for the top and bottom ~~of the box and~~ costs $10 per square foot. Additionally, the length of the box is to be twice its width, and the box must hold a volume of 40 cubic feet. Find the dimensions which will minimize the cost of the box.



$V = lwh = 2w \cdot w \cdot h$

$V = 2w^2h = 40$

$SA = 2(2w^2) + 4(wh)$ $\qquad$ $\text{Cost } SA = \$10(2w \cdot w)2 + \$6(wh)4$

$SA\ \text{Cost} = 40w^2 + 24wh$

$h = \dfrac{20}{w^2}$

$SA\ \text{cost} = 40w^2 + 24wh = 40w^2 + 24w\dfrac{20}{w^2}$

$SA\ \text{cost}' = 80w - \dfrac{480}{w^2} = 0, \quad \dfrac{80w^3 - 480}{w^2} = 0$

$80(w^3 - 6) = 0 \qquad w = \sqrt[3]{6}, \quad l = 2\sqrt[3]{6},$

$h = \dfrac{20}{(\sqrt[3]{6})^2}$

6. (12 points) Answer each of the following questions TRUE or FALSE. (Write the whole word, do not put T or F.)

a) Gasoline is an example of a product whose demand is elastic.

b) If $f''(c) > 0$ and $f'(c) = 0$, then f has a local minimum at $x = c$. T

c) If $f''(c) = 0$, then $(c, f(c))$ must be an inflection point on the graph of f. F

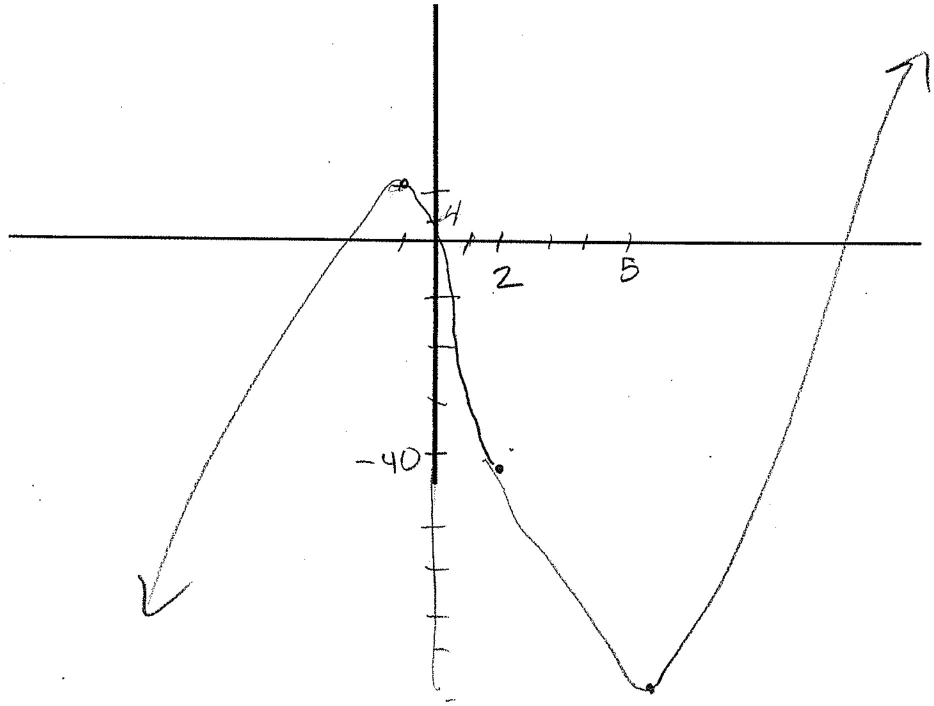
C. up

d) If $\lim_{x\to\infty} f(x) = b$, then $y = b$ is a horizontal asymptote to the graph of f. T

e) It is possible for a function to have two different horizontal asymptotes. T

f) Unit elasticity occurs when the price elasticity of demand, $E(p)$, is equal to 0. F

Graph for Problem 3



Graph for Problem 4