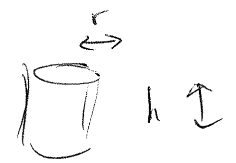
Key to Related Rates Quiz

(1.)



$\frac{dr}{dt} = 3\,m/s$, $\frac{dh}{dt} = 7\,m/s$

Find $\frac{dV}{dt}$ when $r = 6m$ + $h = 5m$

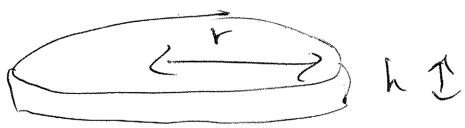
$V = \pi r^2 h$

$$\frac{dV}{dt} = \pi\left(2r\frac{dr}{dt}\cdot h + r^2\frac{dh}{dt}\right)$$

$$\left.\frac{dV}{dt}\right|_{\substack{r=6\\h=5}} = \pi\left(2\cdot 6m\cdot 3\frac{m}{s}\cdot 5m + 6^2m^2\cdot 7\frac{m}{s}\right)$$

$$= 432\pi\, m^3/s$$

(2)



Oil slick $\frac{1}{4}$ in $= \frac{1}{3}$ ft deep, constant height ($h =$)

$V = \pi r^2 h$

$$\frac{dV}{dt} = 2\pi r\frac{dr}{dt}\cdot h = 2\pi\cdot 5ft\cdot 6\frac{ft}{hr}\cdot\frac{1}{3}ft$$

$d = 10\,ft \Rightarrow r = 5ft$

$\frac{dd}{dt} = 12\frac{ft}{hr} \Rightarrow \frac{dr}{dt} = 6\frac{ft}{hr}$

$$= \boxed{20\pi\frac{ft^3}{hr}}$$

(If you left in ~~units~~ dimension of diam d, then $V = \pi\left(\frac{d}{2}\right)^2 h = \frac{\pi d^2\cdot h}{4}$

so $\frac{dV}{dt} = \frac{dV}{dd}\cdot\frac{dd}{dt} = \frac{2\pi d\cdot h}{4}\cdot\frac{dd}{dt}$

$$= \frac{2}{4}\cdot\pi\cdot 10\cdot\frac{1}{3}\cdot 12\,\frac{ft^3}{hr} = \boxed{20\pi\frac{ft^3}{hr}}$$

same answer

Key to Test 2 Review

1

$$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{3}x^{-2/3} - \frac{5}{x^2} = \frac{1}{2\sqrt{x}} - \frac{1}{3x^{2/3}} - \frac{5}{x^2}$$

$$f'(x) = e^x + \pi x^{\pi-1} - \frac{1}{3x}\cdot 3 + 0 = e^x + \pi x^{\pi-1} - \frac{1}{x}$$

$$f'(x) = \frac{1}{x-4}\cdot\frac{1}{\ln 2} - 2(x^3-1)\cdot 3x^2 = \frac{1}{(x-4)\ln 2} \cancel{+\, 6x^2(x^3-1)}$$

$$- 6x^2(x^3-1)$$

$$g'(x) = (3x^2-12x+9)(x^{\pi}) + (x^3-6x^2+9x-3)\,\pi x^{\pi-1}$$

$$g'(x) = \frac{4^x \ln 4 \cdot \log_4 x - 4^x \cdot \frac{1}{x\cdot \ln 4}}{(\log_4 x)^2}$$

$$g'(x) = \frac{1}{6x + e^{3-x^2}} \cdot \left(6 + \left(e^{3-x^2}\right)(-2x)\right)$$

$$= \frac{6 - 2x e^{3-x^2}}{6x + e^{3-x^2}}$$

$$h'(x) = \frac{0\cdot(4-15x^3) - 12(-45x^2)}{(4-15x^3)^2} = \frac{(12)(45x^2)}{(4-15x^3)^2}$$

Implicit problem $\quad 1 = 3x + 2x^2y^3$

Find $\frac{dy}{dx}$: $\quad 0 = 3 + 4xy^3 + 2x^2 \cdot 3y^2 \frac{dy}{dx}$

$$-2x^2 \cdot 3y^2 \frac{dy}{dx} = 3 + 4xy^3$$

$$\frac{dy}{dx} = \frac{3+4xy^3}{-6x^2y^2}$$

When $x = 1$, y is found by plugging $x = 1$ into $1 = 3x + 2x^2y^3$:

$$1 = 3(1) + 2(1^2)y^3 \to y^3 = \frac{-2}{2} = -1$$

$$\to \boxed{y = -1}$$

Tangent line:

$$y - (-1) = \left.\frac{dy}{dx}\right|_{(1,-1)} (x-1)$$

$$y + 1 = \frac{3+4(1)(-1)^3}{-6(1)^2(-1)^2}(x-1)$$

$$y + 1 = \frac{-1}{-6}(x-1)$$

$$y + 1 = \frac{1}{6}(x-1)$$

$Q'(t) = 15 + 2t - 3t^2$ rate of production (units / hr)

$Q'(2) = 15 + 4 - 12 = 7$ units / hr

$Q''(t) = 2 - 6t$ rate of rate of production (units^2 / hr)

$Q''(2) = 2 - 12 = -10$ units^2 / hr

production rate is dropping
(workers are slowing down, or maybe just the machines are!)

• Critical values – Sec 15 – text

#4 p. 129 ~~CB 18~~

a) $f'(x) = 2x - 10 = 0$ at $x = 5$, $f(5) = -33$

b) $f'(x) = x^3 - 6 = 0$ at $x = \sqrt[3]{6}$, $f(\sqrt[3]{6}) = \ldots$
(plug in)

c) $f'(x) = 3x^2 - 2x = 0 \rightarrow x(3x-2) = 0$

$(0,-1), \left(\frac{2}{3}, \frac{-31}{27}\right)$ $x = 0, 2/3$
$f(x) = -1, \left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2 - 1$
~~(calculate)~~

d) $g'(t) = 60t^3 - 30t = 0$

$30t(2t^2 - 1) = 0 \rightarrow t = 0, \pm\sqrt{1/2} = \pm 1/\sqrt{2}$

$\overset{\|}{0}$ $\overset{\|}{0} \rightarrow 2t^2 = 1 \rightarrow t^2 = 1/2$

$(0, -90), \; (1/\sqrt{2}, g(1/\sqrt{2})), \; (-1/\sqrt{2}, g(-1/\sqrt{2}))$

e) $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2\sqrt{x}}$, ~~xxxx~~

f' DNE at $x = 0$, $x = 0 \in$ Dom f

$(0,0)$ is crit pt

f) $f'(x) = \dfrac{(2x-2)(x-3) - (x^2-2x+1)(1)}{(x-3)^2}$

$f' = 2x^2 - 8x + 6 - x^2 + 2x - 1 = 0$

$\Rightarrow x^2 - 6x + 5 = 0 \rightarrow \boxed{x = 5, 1}$
both are in dom f

$(x-5)(x-1) = 0$

~~But~~ f' DNE at $x = 3$, but $3 \notin$ Dom f, so $x = 3$ not crit

g) $f'(x) = \dfrac{1(x^2+x+1) - (x+1)(2x+1)}{\underbrace{(x^2+x+1)^2}_{>0}}$

>0 for all x, so no crit pts under DNE criterion

$f'(x) = x^2 + \cancel{x} + 1 - 2x^2 - \cancel{x} - 2x - 1 = 0$

$-x^2 - 2x = 0$

$-x(x+2) = 0 \longrightarrow x = 0, -2$

$f(0) = 1, \quad f(-2) = -\frac{1}{3}$

h) $f'(x) = 1 + 3 \cdot \frac{2}{3} x^{-1/3} = 1 + \dfrac{2}{x^{1/3}}$

$1 + \dfrac{2}{x^{1/3}} = 0 \longrightarrow 1 = \dfrac{-2}{x^{1/3}}$

$\longrightarrow x^{1/3} = -2 \longrightarrow x = -8 \;\&\; \underbrace{x = 0}_{f' \text{ DNE}}$

$(-8, 4)\ (0, 0)$ crit pts.

i) $f'(x) = \frac{5}{3}(x-1)^{2/3} = 0$ at $x = 1$, $f(1) = 4$

$(1, 4)$

j) $f'(v) = 2v - \frac{4}{2} v^{-1/2} = 2v - \dfrac{2}{\sqrt{v}} = 0$

$\rightarrow 2v = \dfrac{2}{\sqrt{v}} \longrightarrow 4v^2 = \dfrac{4}{v} \rightarrow 4v^3 = 4$

$v^3 = 1 \longrightarrow v = 1, \; f(1) = -3$

& at $v = 0$, $f'(v)$ DNE, so $v = 0$, $f(0) = 0$ is critical pts

p. 92 # 12, 13

12. f, g diff'ble at $x = 3$ and given

$g(3) = 4$, $g'(3) = 5$, $f(3) = 7$, $f'(3) = 6$

Find $h'(3)$ ~~where~~ if $h(x) = f(x)g(x)$

Use operations on derivatives rules:

$h'(x) = (f'g) + (g')(f)\ (x)$ where $x = 3$

$h'(3) = f'(3)g(3) + g'(3)f(3)$

$= 6 \cdot 4 + 5 \cdot 7 = \boxed{59}$

13. f, g diff'ble at $x = 1$ and given

$f(1) = 1$, $f'(1) = 2$, $g'(1) = -3$, $g(1) = \frac{1}{2}$

Find:

a) $(f+g)'(1) = f'(1) + g'(1) = 2 - 3 = \boxed{-1}$

b) $(f-g)'(1) = f'(1) - g'(1) = 2 - {-3} = \boxed{5}$

c) $(2f + 3g)'(1) = 2f'(1) + 3g'(1)$

$= 2 \cdot 2 + 3 \cdot (-3) = \boxed{-5}$

d) $(fg)'(1) = (f'g)(1) + (g'f)(1) = f'(1)g(1) + g'(1)f(1)$

$= (2)(\frac{1}{2}) + (-3)(1) = \boxed{-2}$

13e) $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

so $\left(f/g\right)'(1) = \frac{f'(1)\,g(1) - g'(1)\,f(1)}{g(1)^2}$

$= \frac{(2)(1/2) - (-3)(1)}{(1/2)^2} = \frac{4}{1/4} = \boxed{16}$

Now, suppose $f'(\frac{1}{2}) = -1$

What is $(f \circ g)'(1)$?

$(f \circ g)'(x) = [f(g(x))]'$ by definition of chain rule (on fcn composition)

$= f'(g(x)) \cdot g'(x)$

So $(f \circ g)'(1) = [f(g(1))]'$

$= f'(g(1)) \cdot g'(1)$

$= f'(1/2) \cdot (-3) = (-1)(-3) = \boxed{3}$