Math 108-03 **Fall 2014** **Practice for Exam 2**

1. Which of the following are polynomials? Remember, a polynomial is an expression of the form

$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x^1 + a_0$ where $n \in$ non-negative $\mathbb{Z}$.

a) $-4(x+1)^3 - 4$

b) $7x^{-1} - 2x^4 + \frac{1}{2}$

c) $\sqrt{6}x + 2$

d) $\cos x - 5$

Answers: a, c, f (Note, coefficients can be radicals, as in c; variables can't.)

e) $\dfrac{(x-1)^2}{x^2 - 4x + 1}$

f) -8

2. The equation of a vertical line going through the point (6, –2) is

a) $x = 6$

Answer: a

b) $y = 6$

c) $y = -2$

~~Answer: e~~

d) $y = 6x - 2$

e) $x = -2$

3. Write the equation of the line perpendicular to the line $5x - 2y = 8$ and through the point (1, –1). Give answer in slope-intercept form.

Answer: $y = -\frac{2}{5}x - \frac{3}{5}$

- Put in slope-intercept form to identify slope: $y = \frac{5}{2}x - \frac{8}{2} = \frac{5}{2}x - 4$
- $m_{\text{perpendicular}} = -1/(\frac{5}{2}) = -\frac{2}{5}$
- Substitute the given point and the slope found into the point-slope form of the line:

$$y - (-1) = -\tfrac{2}{5}(x-1)$$
$$y + 1 = -\tfrac{2}{5}x + \tfrac{2}{5}$$
$$y = -\tfrac{2}{5}x + \tfrac{2}{5} - 1$$
$$y = -\tfrac{2}{5}x - \tfrac{3}{5}$$

Give the coordinates of any other point on this line:

ANSWER: Pick any x-value other than the one given, then solve for y using the equation you found. State the results as an ordered pair. (You could even use the y-intercept, say, $x = 0$.)

$y = -\frac{2}{5}x - \frac{3}{5}$ say $x = 0$, so $y = -\frac{3}{5}$ $(0, \frac{3}{5})$

4. According to the rational root theorem, which of the following is NOT a possible rational root of the function?

$$f(x) = 3x^4 - 2x^3 + x^2 - 7x + 12$$

Answer: b

a) $\frac{2}{3}$ b) $-\frac{3}{4}$ c) -12 d) $\frac{4}{3}$ e) none of the above

4 is not a factor of 3 so the ration q/p cannot have $p = 3$.

5. A polynomial of degree n has at most n real zeroes. Because of this it has at most $n - 1$ turns.
https://www.youtube.com/watch?v=9WW0EetLD4Q

Using this information and your understanding of odd or even degree and sign of leading coefficient when dealing with end behavior, match the graph to its possible function.

2 $f(x) = 7x^3 - 21x^2 - 9x + 104$

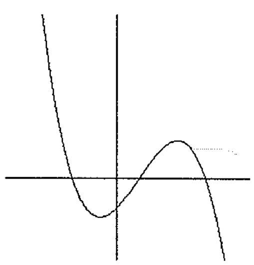
4 $f(x) = x^5 - 8x^4 - 9x^3 + 58x^2 - 164x + 69$

1 $f(x) = -9x^3 + 27x^2 + 54x - 73$

6 $f(x) = -x^5 + 3x^4 + 16x^3 + 2x^2 - 95x - 44$

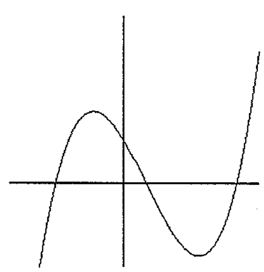
3 $f(x) = x^4 - 6x^3 + 10x^2 - 6x + 9$

7 $f(x) = x^6 - 8x^4 + 12x^2 + 10$



1

$n \geq 3$, l.c. < 0, $f(0) < 0$



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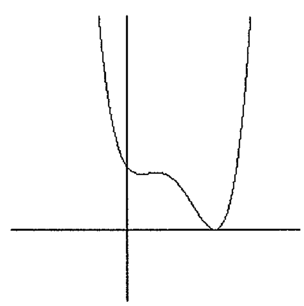
$n \geq 3$, l.c. > 0, $f(0) > 0$



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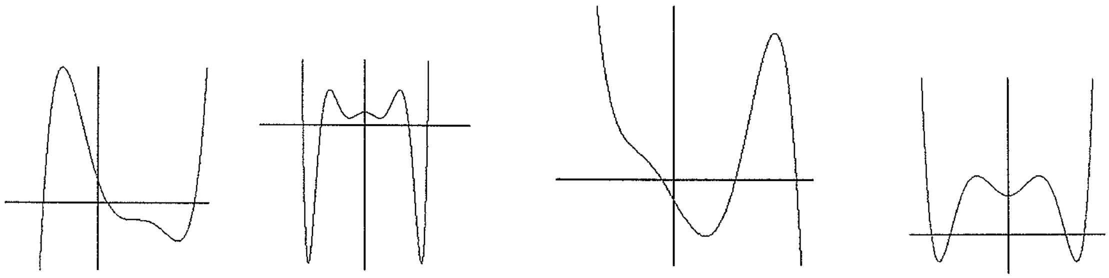
$n \geq 4$, l.c. < 0, $f(0) > 0$



4 5 6 7

$n \geq 5$, l.c. > 0, $f(0) > 0$ $n \geq 8$, l.c. > 0, $f(0) > 0$ $n \geq 4$, l.c. < 0, $f(0) < 0$ $n \geq 6$, l.c. > 0, $f(0) > 0$

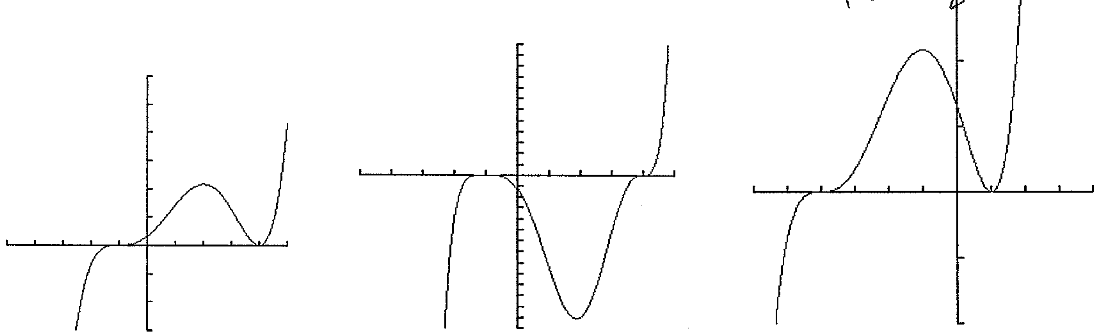
We note that a degree is $\geq$ some value because roots could have multiplicities > 1.

For the graph with no possible function in the selections, make up a function that it could be.

Answer: For graph 5, a possible function is $f(x) = x^8 + 2x^6 - 4x^2 + 2$ (notice that $f(-x) = f(x)$ for this equation)

A better version of this would be to find a minimal degree polynomial with given roots and endpt. behavior. See last quiz.

6. Write possible equations for functions whose graphs are shown.



Roots at –1 and 4 Roots at –1 and 4 Roots at –7 and 2

Answer:

All have 3 turns, so the degree is at most 4, but since shape indicates odd-degree polynomials, we choose 5, 7, etc. for degrees. I went with 5 for each, but adjusted the multiplicities depending on whether the graph crossed or bounced.

* Unless we assume the scale of the y-axis is either very compressed or very expanded, a leading coefficient other than 1 is necessary. Otherwise, the y-intercept would be much larger than the when we expand the

* Not vital – not on test, this bit about a multiplier K.

equation. For example, if the coefficients were 1 in each case, the constant term would be 64 for the first, graph, –64 for the second graph, and the constant term would be 102. A $k < 1$ will compress the graphs.

$f(x) = k(x+1)^3(x-4)^2$ $\qquad$ $f(x) = k(x+1)^2(x-4)^3$ $\qquad$ $f(x) = k(x+7)^1(x-2)^4$

Pay attention to powers of factors, which come from multiplicities needed to get the desired degree.

7. What is true of the slope of line l?

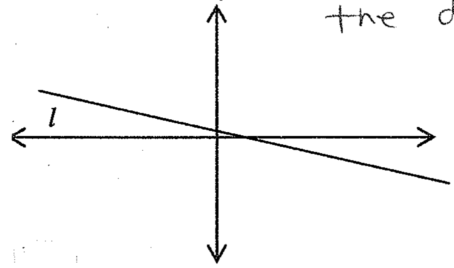
a) $m < -1$

b) $-1 < m < 0$

c) $m = 0$

d) $0 < m < 1$

e) $m > 1$



negative slope

8. Suppose f is a polynomial with the following properties:

f has 1, 3, and 5 as its only roots. As $x \to \infty$, $\quad f(x) \to -\infty$. $\quad$ As $x \to -\infty$, $\quad f(x) \to -\infty$.

A possible equation for f is

a) $f(x) = -(x-1)(x-3)(x-5)$

b) $f(x) = (x-1)^2(x-3)(x-5)$

c) $f(x) = -4(x-1)(x-3)(x-5)^2$ $\qquad$ **Answer: c (*n* odd, l.c. < 0, roots are positive)**

d) $f(x) = -(x+1)^2(x+3)(x+5)$

e) None of the above.

9. The polynomial function $P(x) = 2x^2 + 5x + 1$ has how many real roots?

Answer: Look at the discriminant of the quadratic formula:

$b^2 - ac = 25 - 4(1)(2) = 17 > 0$ hence, it has two real roots.

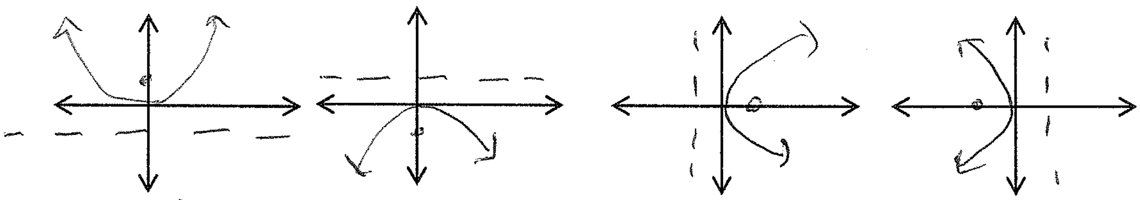
10. For what value(s) of b does the function $P(x) = x^2 + bx + 9$ have one real root (of multiplicity 2)?

Answer: You need the discriminant to be zero, so solve as follows:

$b^2 - 4ac = 0$ $\qquad$ $b^2 - 4(1)(9) = 0$ $\qquad$ $b^2 = 36$ $\qquad$ $\boxed{b = \pm 6}$

Hence, $P(x) = x^2 + 6x + 9$ or $P(x) = x^2 - 6x + 9$.

11. Draw thumbnails of the equations below:



$y = x^2$ (1) $y = -x^2$ (2) $x = y^2$ (3) $x = -y^2$. (4)

From the formula for the parabola $y = \frac{1}{4p}x^2$ and $x = \frac{1}{4p}y$, write the equation of the directrix and the coordinates of focus of each.

Since $\frac{1}{4p} = 1$ in all cases, $p = \frac{1}{4}$.

(1) $y = -\frac{1}{4}$ F: $(0, \frac{1}{4})$

(2) $y = \frac{1}{4}$ F: $(0, -\frac{1}{4})$

(3) $x = -\frac{1}{4}$ F: $(\frac{1}{4}, 0)$

(4) $x = \frac{1}{4}$ F: $(-\frac{1}{4}, 0)$

12. Complete the square to solve the equation:

$$2x^2 - 8x + 7 = 1$$

All steps must be seen.

$$2x^2 - 8x = -6$$
$$2(x^2 - 4x) = -6$$
$$x^2 - 4x = -3$$
$$x^2 - 4x + 4 = -3 + 4$$
$$(x-2)^2 = 1$$
$$x - 2 = \pm\sqrt{1}$$
$$x = 2 \pm \sqrt{1} = 2 \pm 1 \Rightarrow \boxed{x = 3, 1}$$

13. Solve by quadratic formula: $3x^2 + x - 2 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(3)(-2)}}{2(3)}$$
$$= \frac{-1 \pm \sqrt{25}}{6} = \frac{-1 \pm 5}{6}$$

← Don't stop..., must simplify for full crdt.

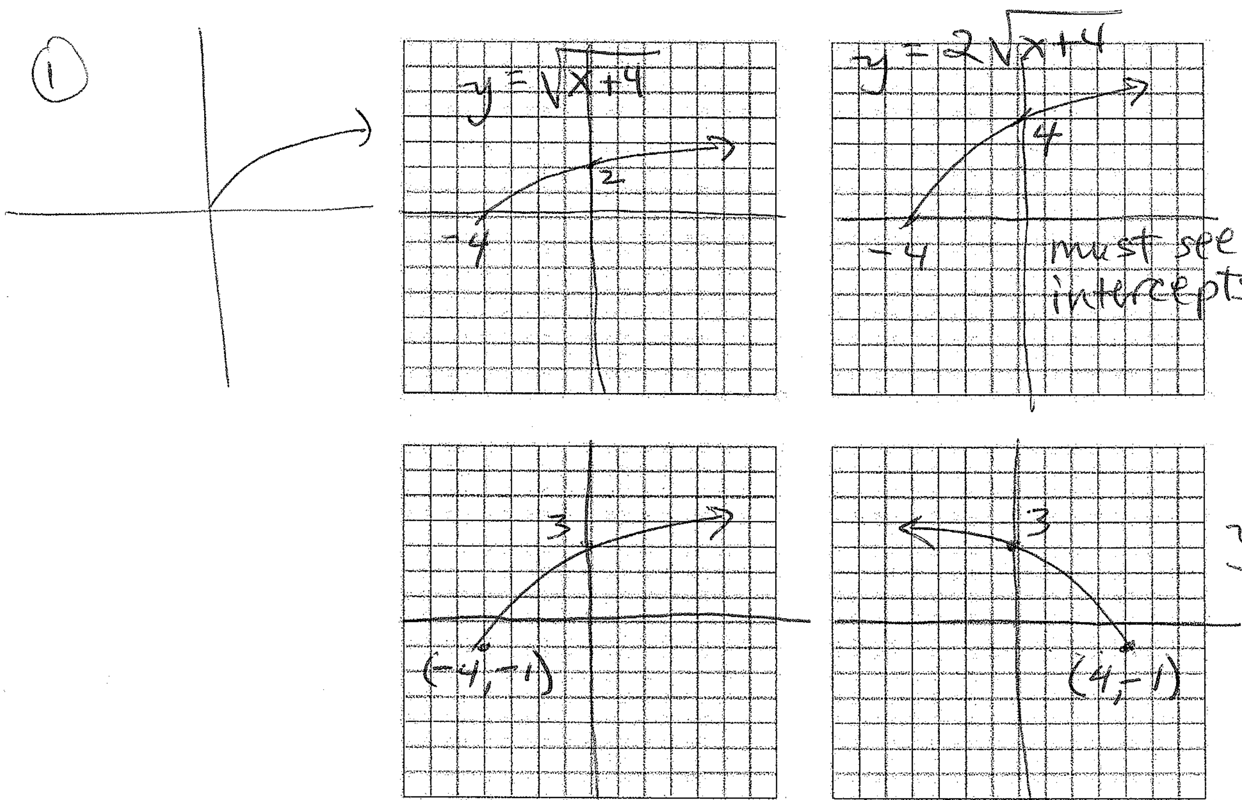
$$= \left(\frac{4}{6}, -\frac{6}{6}\right) = \boxed{\left(\frac{2}{3}, -1\right)}$$

14. Given points $A = (2, -3)$ and $B = (8, 6)$ the length of the line segment AB.

$$d = \sqrt{(2-8)^2 + (-3-6)^2}$$
$$= \sqrt{(-6)^2 + (-9)^2}$$
$$= \sqrt{36+81}$$
$$= \sqrt{117} = \sqrt{9 \cdot 13} = \boxed{3\sqrt{13}}$$

15. Draw the steps in the transformation to $f(x) = 2\sqrt{-x+4} - 1$, from the mother function. Show any x and y intercepts at each step. Label any other relevant point.

Start with $y = \sqrt{x}$, end with reflection across y-axis



$y = 2\sqrt{-x+4} - 1$

Show the important intercepts. In this case the x-intercept is less important than the endpt after the shifts.

16. For the rational function, find the relevant information (intercept, holes, asymptotes, if any) and then draw the graph.

$$f(x) = \frac{3\cancel{(x-1)}(x-2)(x+2)}{2(x-3)^2\cancel{(x-1)}}$$

This is the problem I changed so the x-1 would cancel

Domain: $x \neq 3, 1$ VA: $x = 3$, ~~$x = 1$~~

deg numerator = deg denominator = 3, so HA is ratio of lead coeffs: $y = 3/2$

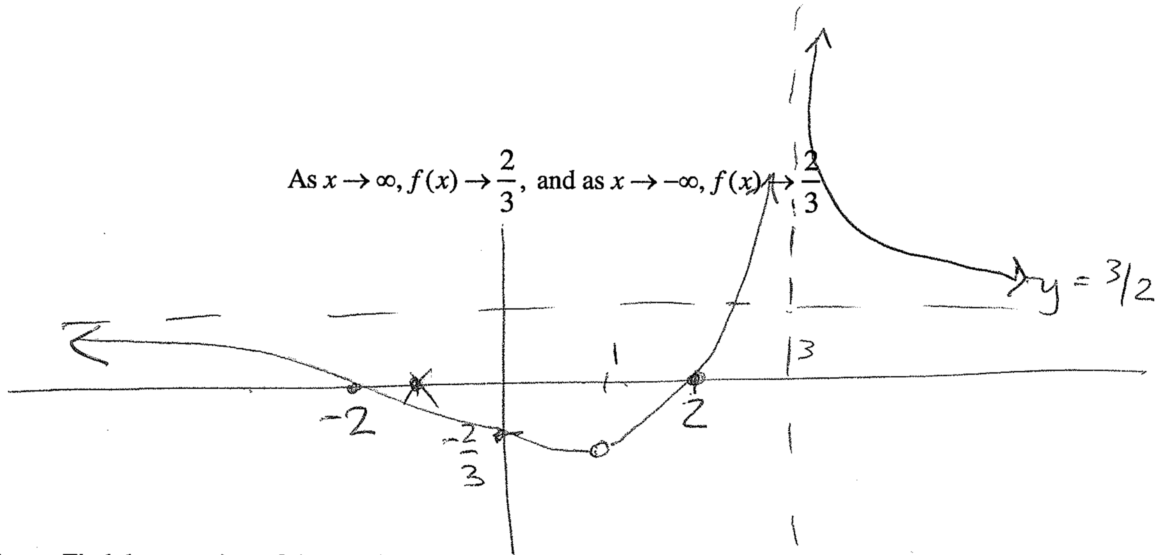
SA: none (requires deg num = deg denom + 1)

~~hole: none (requires a factor in the denominator to disappear by reducing)~~

y-intercept: $f(0) = -12/-18 = 2/3$, x-intercepts: ~~(1,0),~~ (2,0), (–2,0)

hole: at $x = 1$ $f(1) = -\frac{9}{8}$

As $x \to \infty, f(x) \to \frac{2}{3}$, and as $x \to -\infty, f(x) \to \frac{2}{3}$



17. Find the equation of the parabola with a focus at (–3, 4) and a directrix at $y = 2$. (Hint: a sketch of these data tells you the orientation of the parabola, up or down, right or left).

IMPORTANT — $\boxed{y-3=\frac{1}{4}(x+3)^2}$ SEE NOTE ON NEXT PAGE

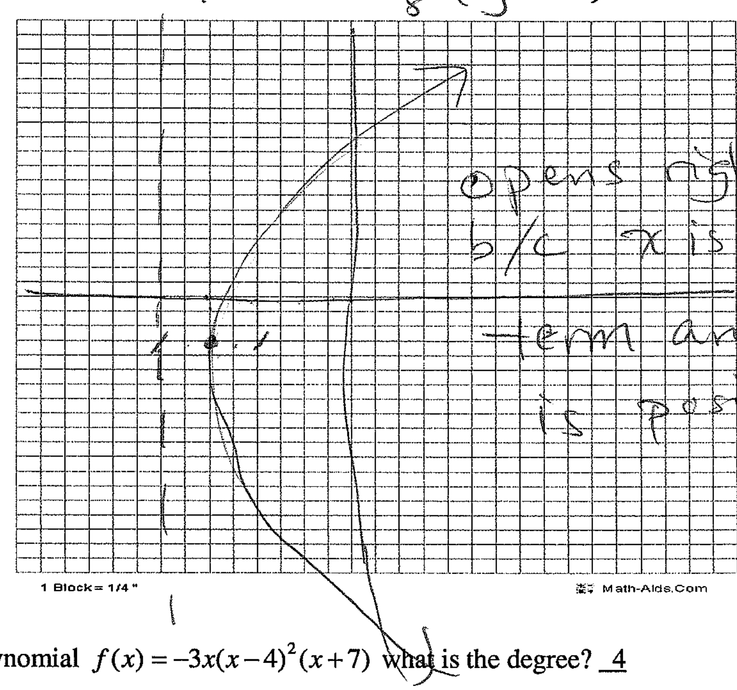
18. Sketch the parabola whose equation is $x=\frac{1}{8}(y+3)^2-6$. Label the vertex and the directrix.

$x+6 = \frac{1}{8}(y+3)^2$

$(h,k)=(-6,-3)$

$\frac{1}{4p} = \frac{1}{8}$

$\Rightarrow p = 2$



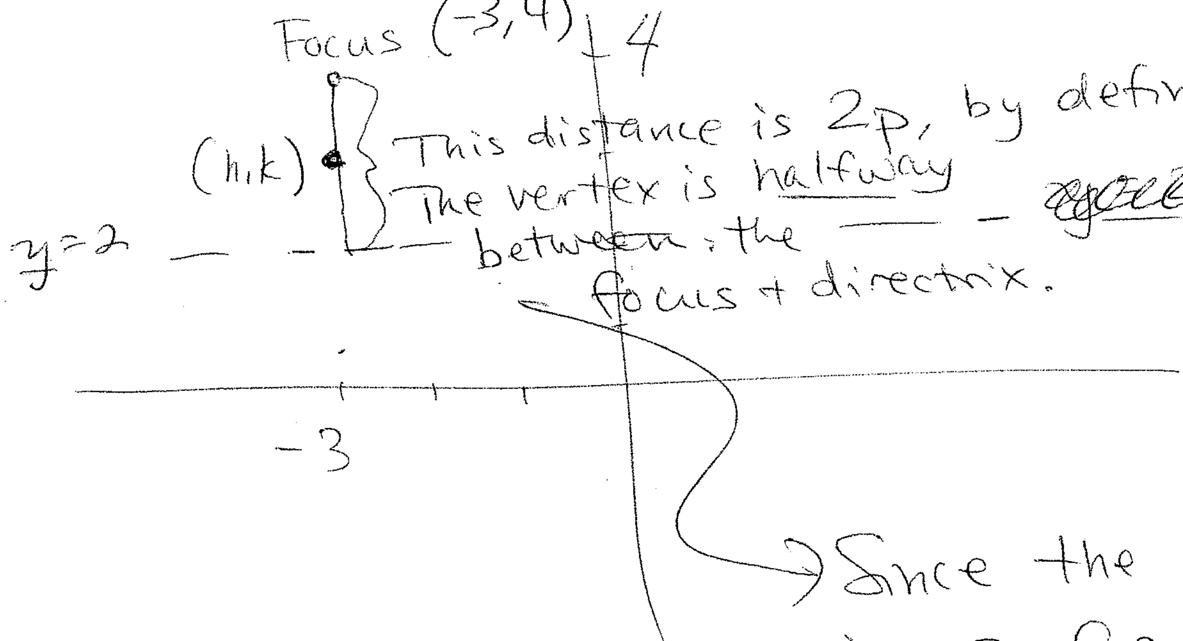
opens right b/c x is the square term and the coeff is positive.

19. For the polynomial $f(x) = -3x(x-4)^2(x+7)$ what is the degree? 4

leading term? $-3x^4$ constant term? 0 (because the expanded equation's lowest degree term is x.

Sketch the polynomial, showing the roots clearly.

You can find p easily from a focus + directrix as follows:



This distance is 2p, by definition. That is, The vertex is halfway ~~equal~~ between the focus + directrix.

Since the distance is 2 from $y=4$ to 2,

$p = \frac{1}{2} \cdot 2 = 1$

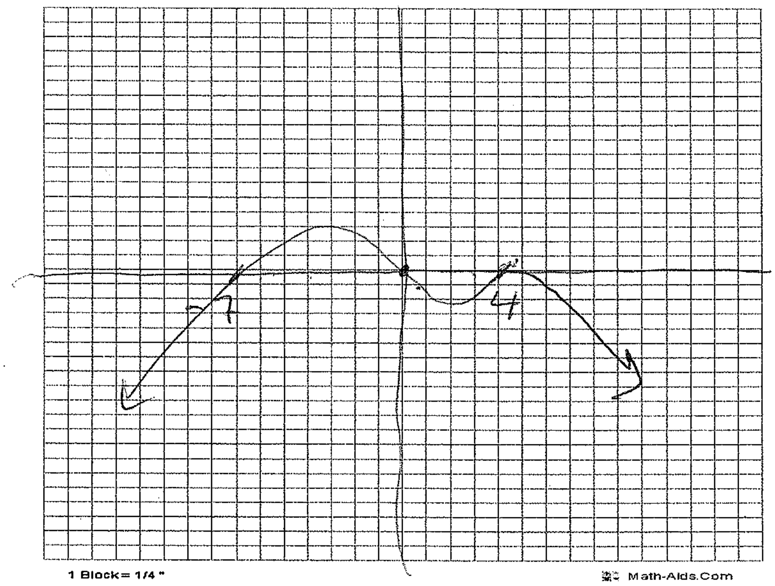
Since $p=1$, the vertex is $(-3,3)$

So, $(h,k) = (-3,3)$, and

$$y-k = \frac{1}{4p}(x-h)^2$$

$$y-3 = \frac{1}{4(1)}(x+3)^2$$

$$\boxed{y = \frac{1}{4}(x+3)^2 + 3}$$



1 Block= 1/4 "

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